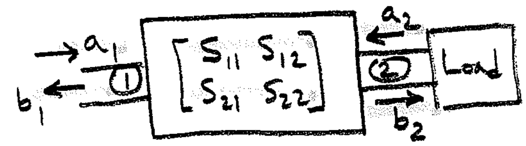
2 additional pages of
Notes on chapter 4

Ex.1 For a two-port circuit, calculate (See also Ex. 4.5 Text p.179)

a. the reflection coefficient Γ_1 at port ① if port ② is connected to a mismatched load of reflection Γ_ℓ

b. the power delivered to this mismatched load in terms of input power $a_1 a_1^*$

Solution a. The circuit diagram is given in Fig. 1



$$b_1 = S_{11} a_1 + S_{12} a_2 \quad (1)$$

$$b_2 = S_{21} a_1 + S_{22} a_2 \quad (2)$$

For the mismatched load of reflection Γ_ℓ, note that $a_2 = \Gamma_\ell b_2$ (3)

Eq. (3) can be rewritten as $b_2 = \frac{a_2}{\Gamma_\ell}$ (4)

Substituting Eq. (4) into Eq. (2) and rearranging terms

$$S_{21} a_1 = \left(\frac{1}{\Gamma_\ell} - S_{22}\right) a_2 = \left(\frac{1 - S_{22}\Gamma_\ell}{\Gamma_\ell}\right) a_2 \quad (5)$$

Substitute Eq. (5) into Eq. (1), we can write

$$\boxed{\Gamma_1 = \frac{b_1}{a_1} = \left[S_{11} + \frac{S_{12} S_{21} \Gamma_\ell}{1 - S_{22}\Gamma_\ell}\right]} \quad (6)$$

b. Power delivered to the mismatched load $= b_2 b_2^* - a_2 a_2^* = b_2 b_2^* (1 - |\Gamma_\ell|^2)$ (7)

Expressing Eq. (2) in terms of a_1, we can write (by using Eq. (5))

$$b_2 = \left[S_{21} + \frac{S_{22} \; S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell}\right] a_1 = \left[\frac{S_{21}}{1 - S_{22}\Gamma_\ell}\right] a_1 \quad (8)$$

Combining Eqs. (7) and (8), we can write

$$\frac{\text{Power delivered to the mismatched load}}{\text{Power input } a_1 a_1^*} = \left|\frac{S_{21}}{1 - S_{22}\Gamma_\ell}\right|^2 \times (1 - |\Gamma_\ell|^2) \quad (9)$$

Ex. 2 Calculate the quantities a. and b. in Ex. 1 for a circuit for which the S-parameters are $\begin{bmatrix} 0.1 & 0.8j \\ 0.8j & 0.2 \end{bmatrix}$ and $\Gamma_\ell = 0.333$

Solution

a. From Eq. (6), $\frac{b_1}{a_1} = \Gamma_1 = \left[S_{11} + \frac{S_{12} \; S_{21} \; \Gamma_\ell}{1 - S_{22} \Gamma_\ell}\right] = -0.1285$

Return loss $RL = 20 \log \left|\frac{b_1}{a_1}\right| = 17.82\,\text{dB}$

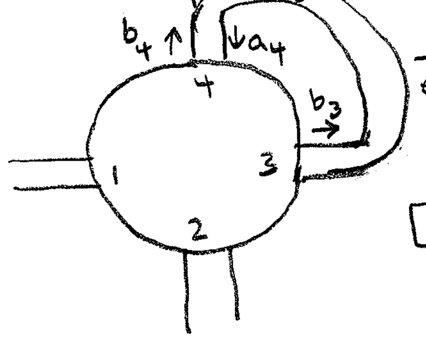
b. From Eq. (9), $\frac{\text{power delivered to the load}}{\text{input power } a_1 a_1^*} = 0.654$

i.e. 65.4 % of the input power can be delivered to this mismatched load

c. For a perfectly matched load connected to the output port ②, $\Gamma_\ell = 0$ and $\frac{\text{power delivered to this perfectly matched load}}{\text{input power}} = |S_{21}|^2 = (0.8)^2 = 0.64$

Similar to Prob. 4.17 p.218 Text

A reciprocal, lossless 4-port network

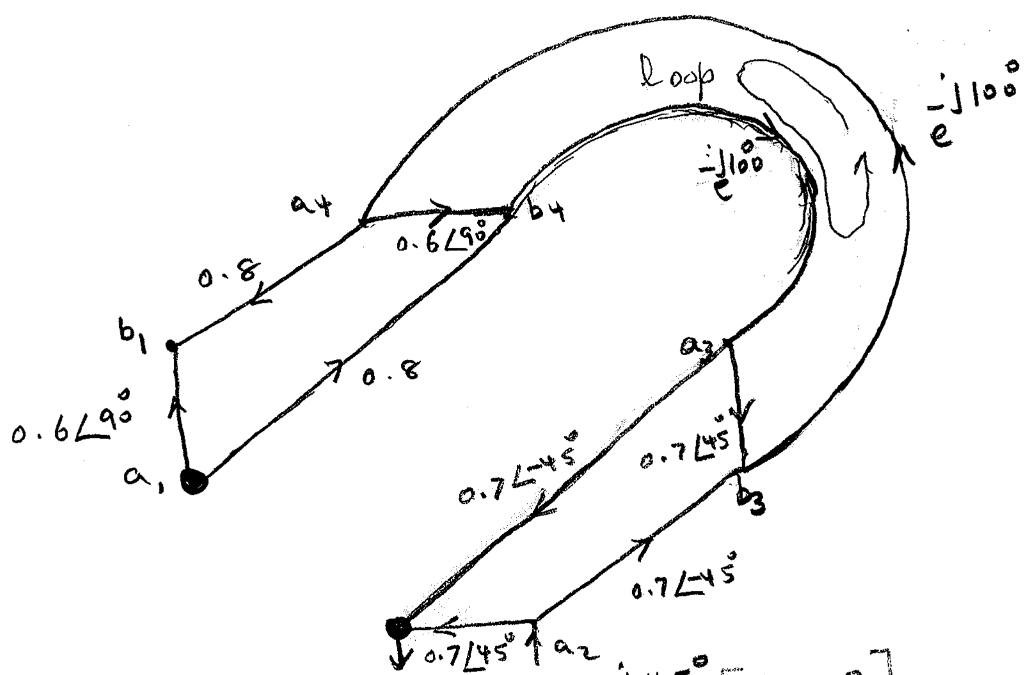


$a_4 = b_3 e^{-j100°}$

$a_3 = b_4 e^{-j100°}$

$$[S] = \begin{bmatrix} 0.6\angle 90° & 0 & 0 & 0.8\angle 0° \\ 0 & 0.7\angle 45° & 0.7\angle -45° & 0 \\ 0 & 0.7\angle -45° & 0.7\angle 45° & 0 \\ 0.8\angle 0° & 0 & 0 & 0.6\angle 90° \end{bmatrix}$$

Note that ports 3 and 4 are connected with a lossless matched transmission line of electrical length 100°



$$\frac{b_2}{a_1} = \frac{\text{path } P_1 = 0.8 \times e^{-j100°} \times 0.7 e^{-j45°}\,[1-0]}{1 - \left[(0.6 e^{+j90°})(e^{-j100°})(0.7 e^{j45°})(e^{-j100°})\right]}$$

$$= \frac{0.56\, e^{-j145°}}{1 - 0.42\, e^{-j65°}}$$

$$\frac{b_2 b_2^*}{a_1 a_1^*} = \frac{(0.56)^2}{(1 - 0.42 e^{-j65°})(1 - 0.42 e^{+j65°})} = \frac{(0.56)^2}{1 + (0.42)^2 - 2 \times 0.42 \cos 65°}$$

$$= \boxed{0.4648}$$

$$IL = -10 \log(0.4648) = -3.33\ \text{dB}$$