Chapter 6
Notes

Chapter 6

A series resonant circuit

A parallel resonant cct (6-1)

$f_o = 3\,GHz$

$Q = 1000$

$f_o = \frac{1}{2\pi\sqrt{LC}}$

$LC = \frac{1}{4\pi^2 f_o^2} = \frac{1}{4\pi^2 \times 9 \times 10^{18}} = 2.81 \times 10^{-21}$

$f_o = \frac{1}{2\pi\sqrt{LC}}$

say $C = 1\,pF$

$L = 2.81\,nH$

$Q = 1000$

$Q = \frac{\omega_o L}{R_s}$ (6.8)

$R_s = \frac{(2\pi \times 10^9 \times 3) \times 2.81 \times 10^{-9}}{Q}$

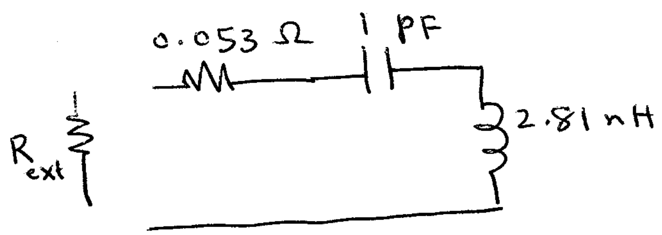
$= 0.053\ \Omega$

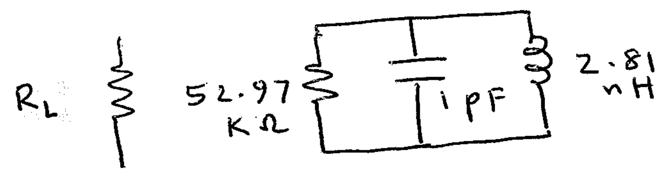
$\frac{R_p}{\omega_o L} = Q$ (6.18)

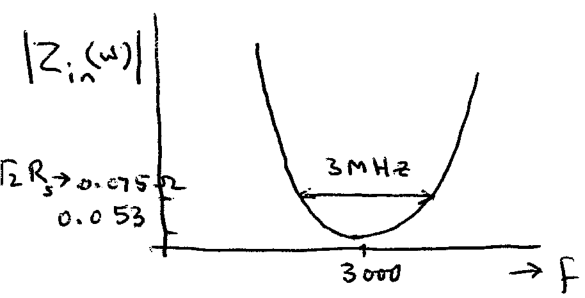
$R_p = 2\pi \times 3 \times 10^9 \times 2.81 \times 10^{-9} \times 10^3$

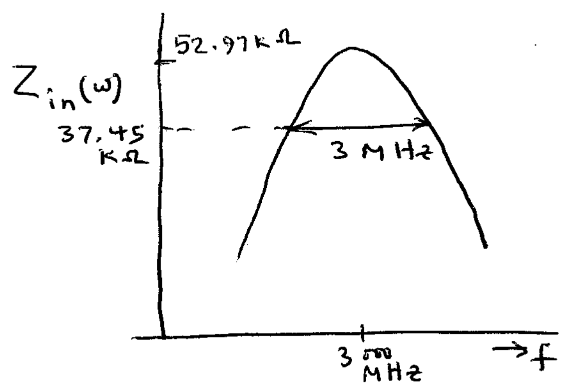
$= 52.97\ K\Omega$

$BW = \frac{f_o}{Q} = \frac{3000}{1000} = 3\ MHz$









$Q_L = \frac{\omega L}{R_{int} + R_{ext}}$

$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_{un}}$ (6.23)

For $R_L = 100\,K\Omega$

$\frac{R_L}{\omega_o L} = Q_e = 1888$

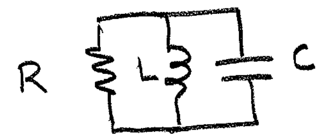
$Q_L = \frac{Q_e Q}{Q + Q_e} = \frac{1888 \times 1000}{2888}$

$= 653.74$

p. 275 Ex. A Quarter Wave Microstrip Resonator made of a short circuited Line (6-2) (6-2)

$$R = \frac{Z_0}{\alpha l} \quad (6.30a) \qquad L = \frac{1}{\omega_0^2 C}$$

$$C = \frac{\pi}{4\omega_0 Z_0} \quad (6.30b)$$


A parallel resonator

$$Q = \frac{\pi}{4\alpha l} \quad (6.31)$$

parallel type resonator

$$l = \lambda_g/4$$

from pp. 144, 145

For $Z_0' = 100\ \Omega$ $\qquad A = 2.123 > 1.52$

$$\frac{W}{d} = 0.896$$

$\epsilon_r = 2.3$ $\qquad f = 3\ GHz$ $\qquad \tan\delta = 10^{-3}$ $\qquad d = 1/32'' = 0.793\ mm$

$W = 0.711\ mm$

(3.195) $$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\,\frac{1}{\sqrt{1 + 12\frac{d}{W}}} = 1.82$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_e}} = \frac{10}{\sqrt{1.82}} = 7.41\ cm$$

$$l = \lambda_g/4 = 1.852\ cm$$

(3.198) $$\alpha_d = \frac{k_0 \epsilon_r(\epsilon_e - 1)\tan\delta}{2\sqrt{\epsilon_e}\,(\epsilon_r - 1)} = \frac{2\pi}{10\times10^{-2}} \times \frac{2.3\times0.82\times10^{-3}}{2\sqrt{1.82}\times1.3} = 3.38\times10^{-2}\ Np/m$$

(3.199) $$\alpha_c = \frac{R_s}{Z_0 W} = 1.988\sqrt{\frac{\cancel{f_{MHz}}\ 3000}{\underbrace{5.817}_{\sigma_{Cu}}\times\cancel{10^7}\ 10}} \times \frac{1}{100\times0.711\times\cancel{10^{-3}}} = 20.08\times10^{-2}\ Np/m$$

$$R_s = \frac{1}{\sigma\delta_s}$$

$$\alpha_T = \alpha_c + \alpha_d = 23.46\times10^{-2} = 0.2346\ Np/m$$

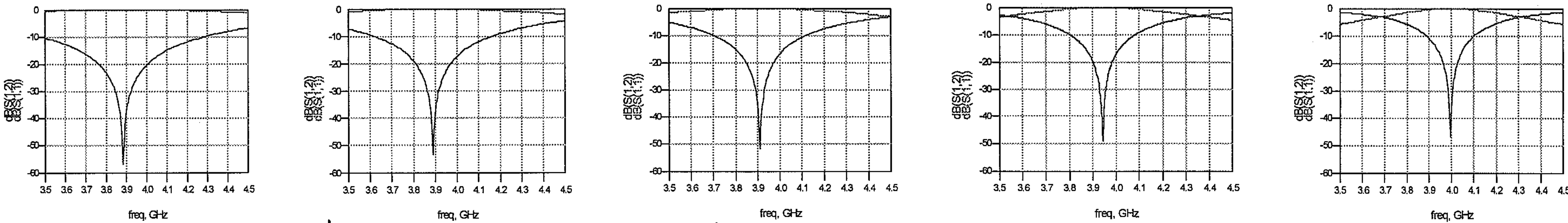
$$R = \frac{Z_0}{\alpha l} = \frac{100}{0.2346\times1.852\times10^{-2}} = 2.3\times10^{4} = 23\ K\Omega$$

$$C = \frac{\pi}{4 W_0 Z_0} = \frac{\pi}{4\times3\times10^9\times100} = 2.62\ pF$$

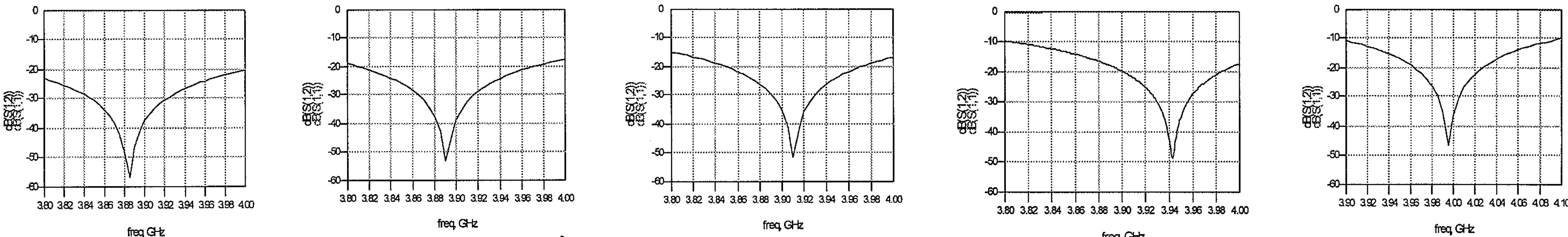
$$L = \frac{1}{W_0^2 C} = \frac{1}{4\pi^2\times10^{18}\times9\times2.62\times10^{-12}} = 1.074\ nH$$

$$Q_{unloaded} = \frac{\pi}{4\times\alpha l} = \frac{\pi}{4\times0.2346\times1.852\times10^{-2}} = 180.8$$

Compare this to a Q of 526 calculated for a $Z_0 = 50\ \Omega$ on p. $\frac{1}{2}$ microstrip reson



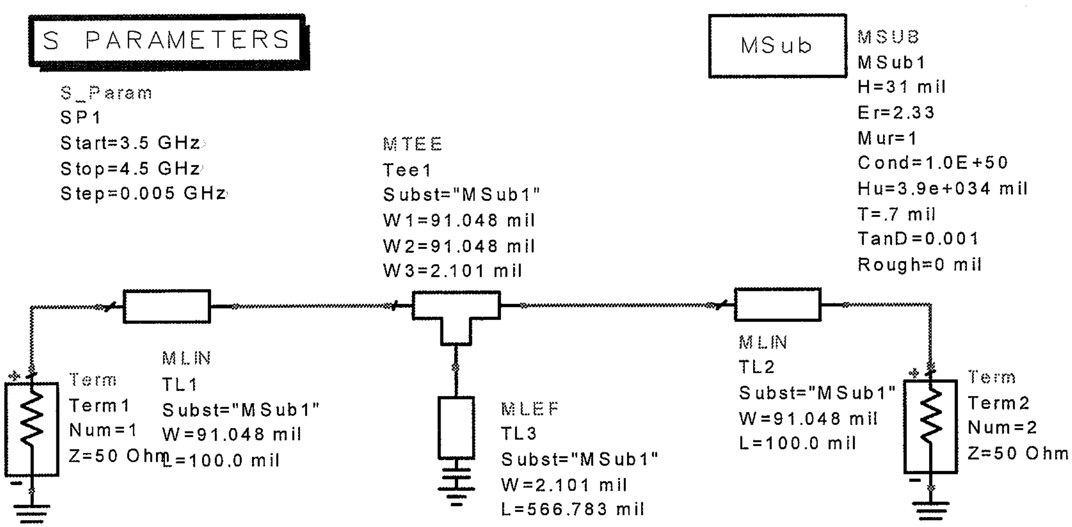
Broader scan from 3.5 – 4.5 GHz



Zoom scan from 3.80 to 4.00 GHz

Zo_stub = 50 ohm	Zo_stub = 75 ohm	Zo_stub = 100 ohm	Zo_stub = 150 ohm	Zo_stub = 200 ohm
W = 91.048 mil	W = 46.084 mil	W = 25.260 mil	W = 7.771 mil	W = 2.101 mil
$\lambda_g/4 \rightarrow$ L = 525.039 mil	L = 536.606 mil	L = 545.283 mil	L = 557.087 mil	L = 566.783 mil

Note that the resonance frequency is lower as broader (lower impedance) stubs are used.



p.292 **Excitation of Resonators**

For $R_{L \to ext} = R_{int}$ $\qquad Q_{ext} = Q_{unloaded}$

Critically coupled

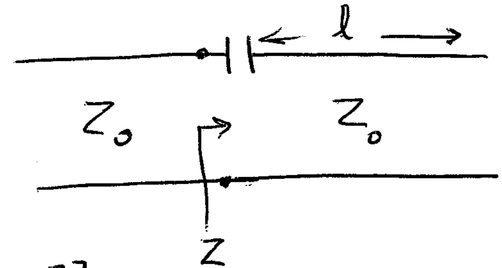
We can define (6.76)

$$g = \frac{Q_u}{Q_e} = \frac{R_{int}}{R_{ext}} \quad \text{for a parallel resonator}$$

$$= \frac{R_{ext}}{R_{int}} \quad \text{for a series resonator}$$

			for a parallel resonator
$g < 1$	under coupled	$Q_u < Q_{ext}$	$R_{int} < R_{ext}$
$g = 1$	Critically coupled	$Q_u = Q_{ext}$	$R_{int} = R_{ext}$
$g > 1$	Over coupled	$Q_u > Q_{ext}$	$R_{int} > R_{ext}$.

p.292 For a gap coupled microstrip resonator



$$z = \frac{Z}{Z_0} = \frac{-j\left[\frac{1}{\omega C} + Z_0 \cot \beta l\right]}{Z_0}$$

$$= -j\left(\frac{\tan \beta l + b_c}{b_c \tan \beta l}\right) \quad (6.77)$$

$b_c = Z_0 \omega C$ is the normalized susceptance of the coupling Capacitor C

Resonance occurs for $z = 0$ or when

$$\tan \beta l + \omega C Z_0 = \tan \beta l + b_c = 0 \quad (6.78)$$

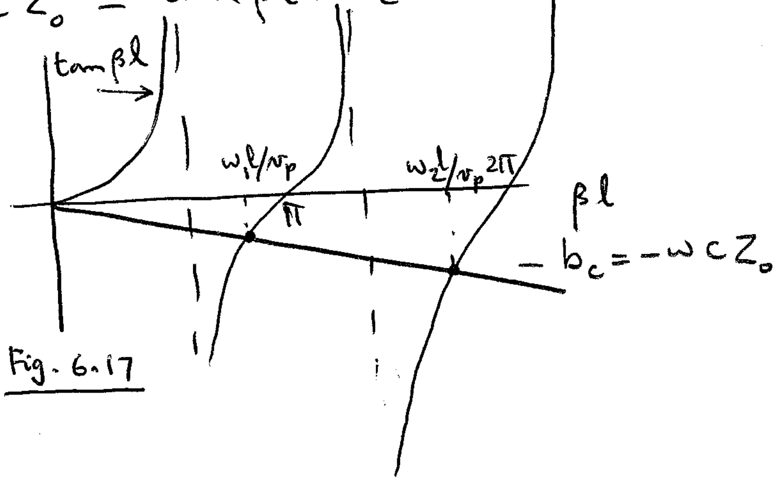


Fig. 6.17

At resonance

$$R = \frac{Z_0 \pi}{2 Q b_c^2}$$

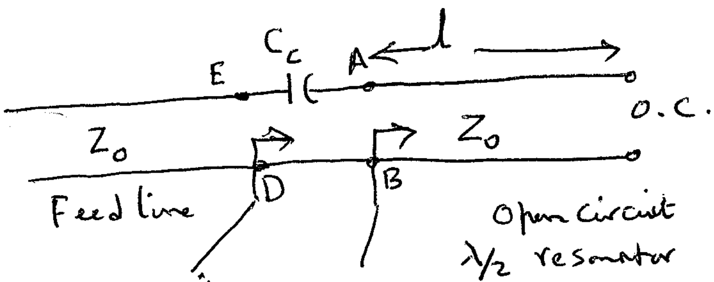
For critical coupling $R_{se} = Z_0$

or $b_c = \omega C Z_0 = \sqrt{\frac{\pi}{2Q}}$

pp. 292-295

A Gap-Coupled Microstrip Resonator (pp. 292 – 295 Text)

Also See Ex. 6.6 p. 295

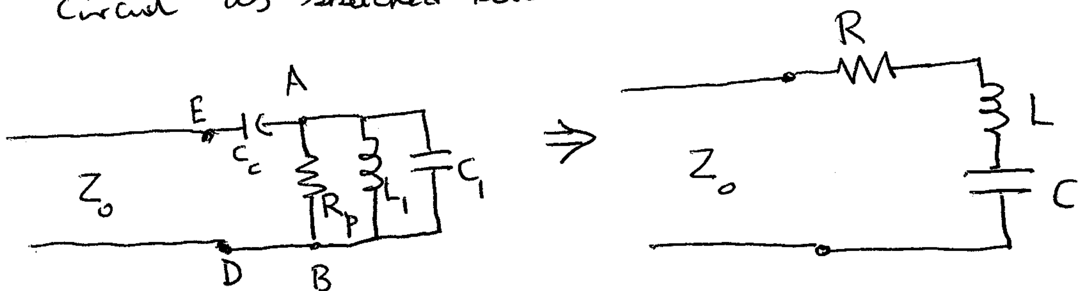


$Z_{ED} = \frac{-j}{\omega C_c} + Z_{AB}$; $Z_{AB} = -j\, Z_o \cot \beta l$

$$z = \frac{Z_{ED}}{Z_o} = -\frac{j}{\omega C_c Z_o} - j \cot\beta l = -j\left(\frac{\tan \beta l + b_c}{b_c \tan \beta l}\right) \quad (6.77)$$

where $b_c = Z_o \omega C_c$ is the normalized susceptance of the coupling capacitor C_c

The coupling capacitor acts as an impedance inverter and converts the equivalent parallel resonant circuit between terminals A B to a series resonant circuit as sketched below



Eq. 6.81 is similar to the expression (6.9) for a series resonant circuit.

From Eq. 6.81

$$z(\omega) = \frac{\pi}{2Q\, b_c^2} + j\frac{\pi\, \Delta\omega}{\omega_1\, b_c^2} \quad \text{of the form} \quad \frac{R}{Z_o} + \frac{j\, 2L\, \Delta\omega}{Z_o} \quad (6.9)$$

For critically coupled ~~uncoupled~~ ~~resonant~~ resonator

$$g = \frac{R}{Z_o} = 1 = \frac{\pi}{2Q\, b_c^2}$$

This gives $b_c = \omega C_c Z_o = \sqrt{\frac{\pi}{2Q}}$ (6.82)

$$g = \frac{R_{ext}}{R_{int}} = \frac{Z_o}{R} = \frac{2Q\, b_c^2}{\pi} \quad (6.83)$$

For the new EQVt ckt

$$\frac{R}{Z_o} = \frac{\pi}{2Q\, b_c^2} \Rightarrow R = \frac{\pi Z_o}{2Q\, b_c^2}$$

$$\frac{2L}{Z_o} = \frac{\pi}{\omega_1\, b_c^2} \Rightarrow L = \frac{\pi Z_o}{2\omega_1 b_c^2}$$

$$\omega_1 = \sqrt{\frac{1}{LC}} \Rightarrow C = \frac{1}{\omega_1^2 L}$$

The new resonant frequency ω_1 is obtained from the transcendental Eq.

$$\tan\left(\frac{\omega l}{v_p}\right) + b_c = \tan\left(\frac{\omega l}{v_p}\right) + \omega C_c Z_o = 0 \quad (6.78)$$