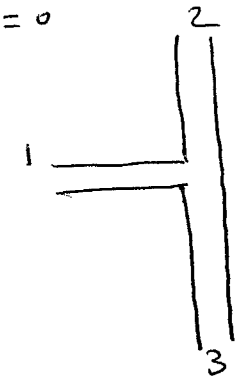
Notes on Chapter 7

Chapter 7

A 3-port, lossless, reciprocal network such as a T-junction
Cannot be matched at all ports

Say it can be matched at port 1; $S_{11} = 0$
Note that ports 2 and 3 are symmetrical



$$[S] = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & S_{22} & S_{23} \\ 1/\sqrt{2} & S_{23} & S_{22} \end{bmatrix} \quad (1)$$

From Columns 2 or 3

$$\frac{1}{2} + |S_{22}|^2 + |S_{23}|^2 = 1 \quad (2)$$

Therefore,

$$|S_{22}|^2 + |S_{23}|^2 = 1/2 \quad (3)$$

Also from Columns 1, 2 or 1, 3

$$\frac{1}{\sqrt{2}} S_{22}^* + \frac{1}{\sqrt{2}} S_{23}^* = 0 \quad (4)$$

Thus $S_{22} = -S_{23}$ (5)

From (3) and (5)

$$2|S_{22}|^2 = 2|S_{23}|^3 = 1/2$$

$$|S_{22}| = 1/2 = |S_{23}|$$

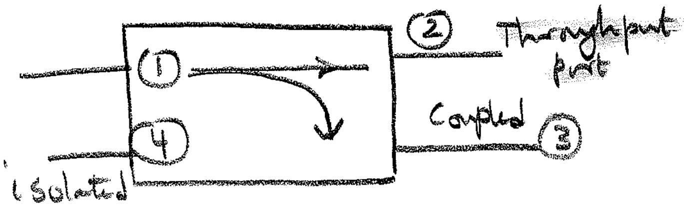
From Eq. (5) $\quad S_{22} = 1/2$; $S_{23} = -1/2$

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ \frac{1}{\sqrt{2}} & 1/2 & -1/2 \\ \frac{1}{\sqrt{2}} & -1/2 & 1/2 \end{bmatrix}$$

This matrix all of the conditions for a loss less, reciprocal network.

A Four-port Circuit (Lossless, reciprocal cct.) (7-2

p. 312



$S_{41} = S_{32} = 0$

$S_{21} = S_{34} = \alpha$

$$[S] = \begin{bmatrix} 0 & \alpha & S_{13} & 0 \\ \alpha & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & \alpha \\ 0 & S_{24} & \alpha & 0 \end{bmatrix}$$

Two possibilities

1. $S_{31} = S_{42} = j\beta$
2. $S_{42} = -S_{31} = -\beta$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

Symmetrical coupler

90° phase shift for outputs at ports 2 and 3

For a lossless network $\alpha^2 + \beta^2 = 1$ (7.19)

$$[S] = \begin{bmatrix} 0 & \alpha & -\beta & 0 \\ \alpha & 0 & 0 & \beta \\ -\beta & 0 & 0 & \alpha \\ 0 & \beta & \alpha & 0 \end{bmatrix}$$

Antisymmetric coupler

180° phase shifts for output ports 2 and 3

$C = 0.1 \rightarrow$ 20 dB coupler

$0.0316 \rightarrow$ 30 dB coupler

Coupling $C = 10 \log_{10}\left(\frac{P_1}{P_3}\right) = -20 \log \beta$ dB (7.20 a)

Directivity $D = 10 \log\left(\frac{P_3}{P_4}\right) = 20 \log \frac{\beta}{|S_{14}|}$ dB (7.20 b)

Isolation $= I = 10 \log\left(\frac{P_1}{P_4}\right) = -20 \log |S_{14}| = C + D$ (7.20 c)

p. 313 For $\beta = \alpha = \frac{1}{\sqrt{2}}$ — 3 dB Hybrid (equal power divider)

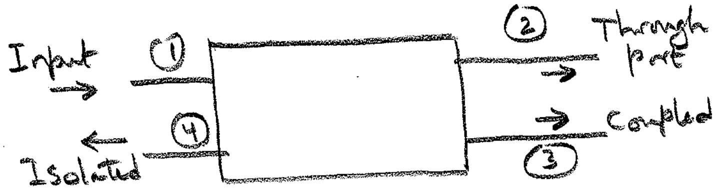
90° hybrid — branch line coupler p. 333

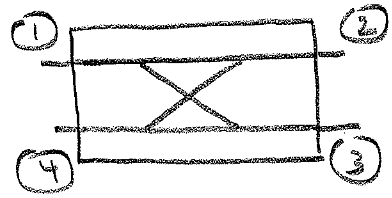
180° hybrid — ring hybrid p. 353

pp. 311 – 314

Four port Directional Couplers

(7-3





All ports can be matched

Note $S_{21} = S_{12}$ $\quad S_{34} = S_{43}$

$S_{21} = S_{34}$, $S_{14} = S_{23}$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$S_{13} S_{23}^* + S_{14} S_{24}^* = 0 = S_{12} S_{13}^* + S_{24} S_{34}^* = S_{13} S_{14}^* + S_{23} S_{24}^*$$

$|S_{13}|^2 = 1 - |S_{12}|^2 = 1 - \alpha^2$

Possible solutions

1. $S_{14} = 0 = S_{23}$; $\quad S_{12} = \alpha$; $\quad S_{13} = j\beta = S_{24}$

Symmetric coupler

$S_{13} = S_{24}$ (both are imaginary)

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

$\boxed{\alpha^2 + \beta^2 = 1}$ (7.17) p. 312

2. $S_{14} = 0$; $\quad -S_{13} = S_{24}$

Antisymmetric coupler

$S_{24} = -S_{13}$ (both are real)

$$[S] = \begin{bmatrix} 0 & \alpha & -\beta & 0 \\ \alpha & 0 & 0 & \beta \\ -\beta & 0 & 0 & \alpha \\ 0 & \beta & \alpha & 0 \end{bmatrix}$$

$\boxed{\alpha^2 + \beta^2 = 1}$ (7.18) p. 312

$IL = -20 \log |S_{12}|$

$RL| = -20 \log |\Gamma|$

Eq. 2.38 $= -20 \log |S_{11}|$

p. 313

1. Coupling $C = 10 \log_{10} \frac{P_1}{P_3} = -20 \log \beta$ dB

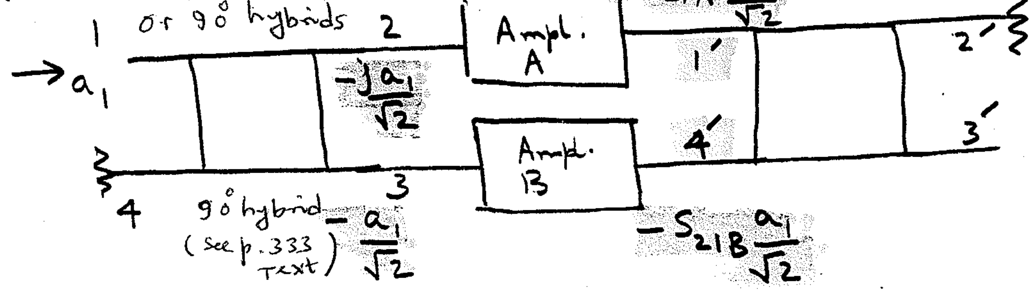
3. Isolation $I = 10 \log_{10} \frac{P_1}{P_4} = -20 \log |S_{14}|$ dB $= C + D$

2. Directivity $D = 10 \log_{10} \frac{P_3}{P_4} = -20 \log \frac{\beta}{|S_{14}|}$ dB

pp. 562, 563 Text

Use of a branch line coupler for a balanced amplifier

See also p. 562 Text



$$\begin{bmatrix} b_1' \\ b_2' \\ b_3' \\ b_4' \end{bmatrix} = -\frac{1}{\sqrt{2}} \underbrace{\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}}_{\text{Eq. 7.61 p. 333}} \begin{bmatrix} -S_{21A}\, j\frac{a_1}{\sqrt{2}} \\ 0 \\ 0 \\ -S_{21B}\frac{a_1}{\sqrt{2}} \end{bmatrix}$$

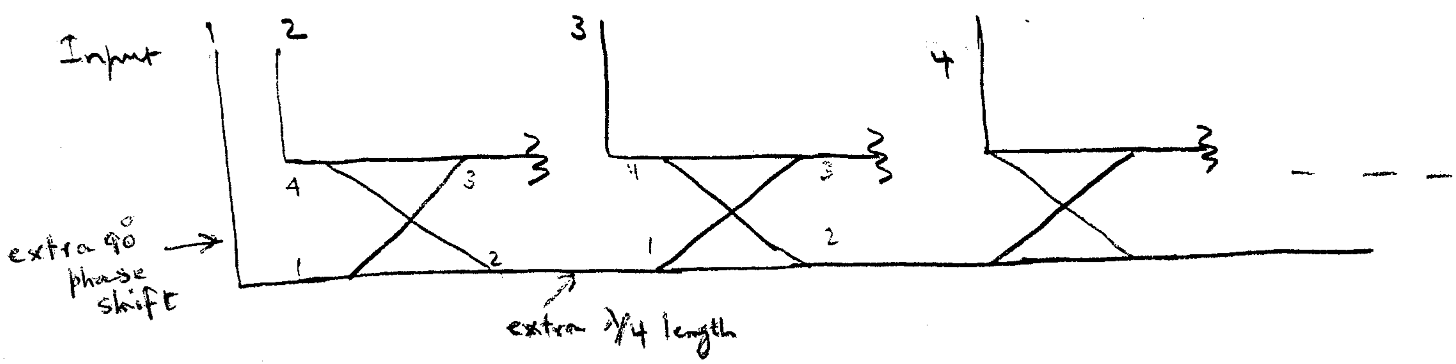
$$b_1' = b_4' = 0$$

See Eqs. 11.62, 11.63, 11.64

$$b_2' = -\frac{1}{\sqrt{2}}\left\{S_{21A} - S_{21B}\right\}\frac{a_1}{\sqrt{2}} \simeq 0$$

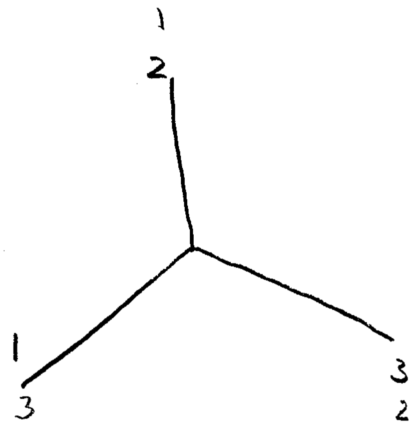
$$b_3' = +j/\sqrt{2}\left\{S_{21A} + S_{21B}\right\}\frac{a_1}{\sqrt{2}}$$

A chain or a serial combiner or dividers



S-parameters of a (angularly) Symmetric Lossless, 3-port reciprocal Network

A Y-junction



$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{23} \\ S_{13} & S_{23} & S_{11} \end{bmatrix}$$

$$S_{12} = S_{31} = S_{13} = S_{23}$$

output port, input port

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{bmatrix}$$

$$|S_{11}|^2 + 2|S_{12}|^2 = 1 \qquad \rightarrow \qquad 9\,S_{11}^2 = 1$$

$$S_{11} = \pm \tfrac{1}{3},\ \pm \tfrac{j}{3}$$

$$S_{12} = \mp \tfrac{2}{3},\ \mp \tfrac{2j}{3}$$

$$S_{11} S_{12}^* + S_{12} S_{11}^* + |S_{12}|^2 = 0$$

Assuming that S_{11}, S_{12} are real

$$2 S_{11} S_{12} + S_{12}^2 = 0 \Rightarrow S_{12} = -2 S_{11}$$

$$S = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$e^{j\theta_1} = j^{1/2};\quad e^{j\theta_3/2} = e^{j\theta_2} = 1$$

Also possible

$$S = \begin{bmatrix} \frac{j}{3} & -\frac{2}{3}j & \frac{2}{3}j \\ -\frac{2}{3}j & \frac{1}{3}j & \frac{2}{3}j \\ \frac{2}{3}j & \frac{2}{3}j & \frac{j}{3} \end{bmatrix}$$

$$\theta_1 = \theta_2 = \theta_3 = 45^\circ$$

$$e^{2j\theta_1} = j = e^{j(\theta_1+\theta_2)}$$