Chapter 5

Conjugate Matching by lumped element circuits and by transmission lines

USE OF LUMPED ELEMENTS FOR MATCHING A LOAD (See pp.222–227 Text)

Example: A Matching Circuit for an Antenna of a Cellular Telephone

Topology 1

The antenna impedance is given to be 50 - j20Ω. The solid-state source to which this impedance is to be matched has an internal impedance, say 15 + j130Ω. A possible matching circuit is sketched as follows:

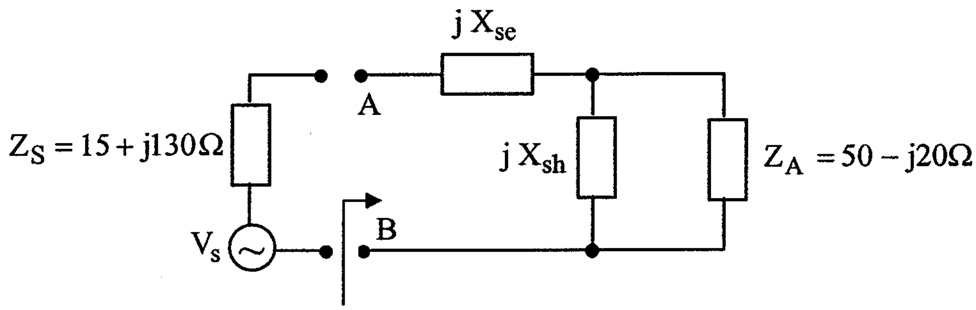

Fig. 1

For maximum power transfer to the antenna

$$Z_{AB} = Z_S^* = 15 - j130\Omega \tag{1}$$

$$\begin{aligned} Z_{AB} &= \frac{jX_{sh}(50 - j20)}{50 + j(X_{sh} - 20)} + jX_{se} \\ &= \frac{(20\,X_{sh} + j50\,X_{sh})\left[50 - j(X_{sh} - 20)\right]}{(50)^2 + (X_{sh} - 20)^2} + j\,X_{se} \\ &= 15 - j130\Omega \end{aligned} \tag{2}$$

Equating real parts on both sides of Eq. 2

$$1000\,X_{sh} + 50\,X_{sh}(X_{sh} - 20) = 15\left[2900 + X_{sh}^2 - 40\,X_{sh}\right]$$

$$35\,X_{sh}^2 + 600\,X_{sh} - 43{,}500 = 0 \tag{3}$$

$$X_{sh} = \frac{-600 \pm \sqrt{(600)^2 + 4 \times 35 \times 43{,}500}}{70} = \frac{-600 \pm 2540}{70}$$

$$= -44.86; \quad +27.71\Omega$$

Taking the capacitive shunt reactance -j44.86Ω and equating the imaginary parts on both sides of Eq. 2, we get

$$X_{se} = -j104.6\Omega = \frac{1}{j\omega C_{se}}$$

$$|X_{sh}| = \frac{1}{\omega C_{sh}} = 44.86\Omega$$

For f = 900 MHz

$$C_{sh} = 3.94 \text{ pF}$$

$$|X_{se}| = \frac{1}{\omega C_{se}} = 104.6\Omega$$

$$C_{se} = 1.69 \text{ pF}$$

The matching circuit for Topology 1 is as follows:

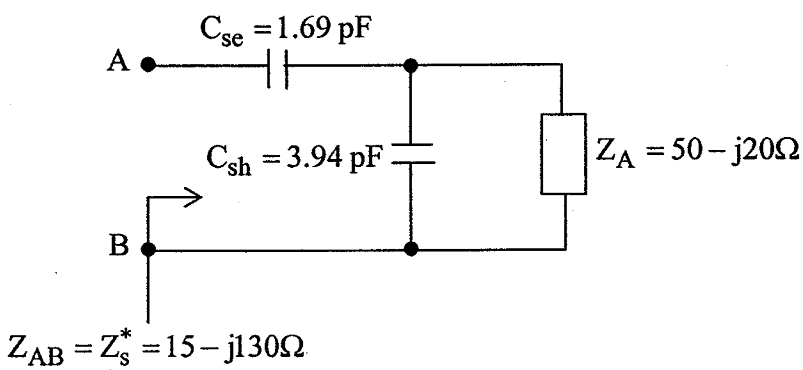

Fig. 2

Topology 2

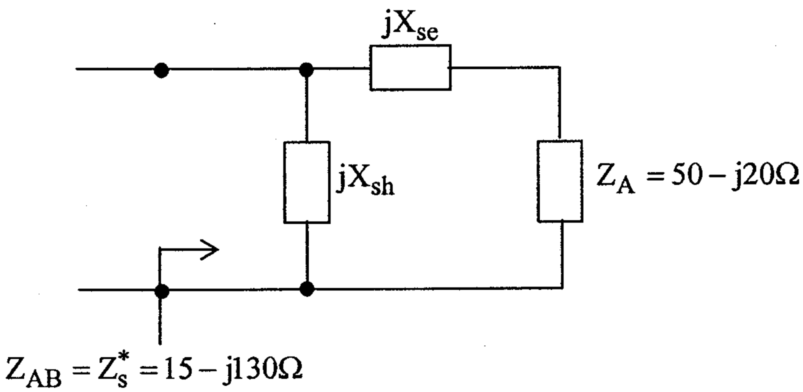

Fig. 3

This problem may be easier to solve in terms of admittances

$$Y_{AB} = \frac{1}{15 - j130} = \frac{15 + j130}{(15)^2 + (130)^2} = \frac{1}{50 + j(X_{se} - 20)} + \frac{1}{jX_{sh}} \quad (4)$$

Equating real parts

$$\frac{15}{17{,}125} = \frac{50}{(50)^2 + (X_{se} - 20)^2} \quad (5)$$

$$2500 + (X_{se} - 20)^2 = \frac{50 \times 17125}{15} = 57{,}083$$

$$X_e = 253.6; \qquad -213.6\Omega$$

Taking the sines inductance

$$X_{se} = 253.6\Omega \Rightarrow L_{se} = 44.85 \text{ nH}$$

$$X_{sh} = -85.59\Omega \Rightarrow C_{sh} = 2.06 \text{ pF}$$

Implications for Power Transfer

a. Without conjugate matching, for an oscillator voltage $V_s = 2V$ RMS power

$$\text{Power transferred to the load} = I_{rms}^2 R_A = \frac{(2)^2}{(15 + 50)^2 + (130 - 20)^2} \times 50 = 12.25 \text{ mW}$$

b. With conjugate matching

$$\text{Power transferred to the load} = I_{rms}'^{\,2} \operatorname{Re}|Z_s^*| = \frac{(2)^2}{(15 + 15)^2} \times 15 = 66.7 \text{ mW}$$

Needed for 600 mW power transferred to the load

$$V_s = 6V \text{ RMS}$$

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A REACTIVE THREE-ELEMENT CIRCUIT FOR ANTENNA MATCHING

A reactive three-element network is a versatile circuit for matching power onto the antenna.

To illustrate the procedure, let us look at the circuit of Fig. 1. The antenna equivalent impedance is $R_a + jX_a$.

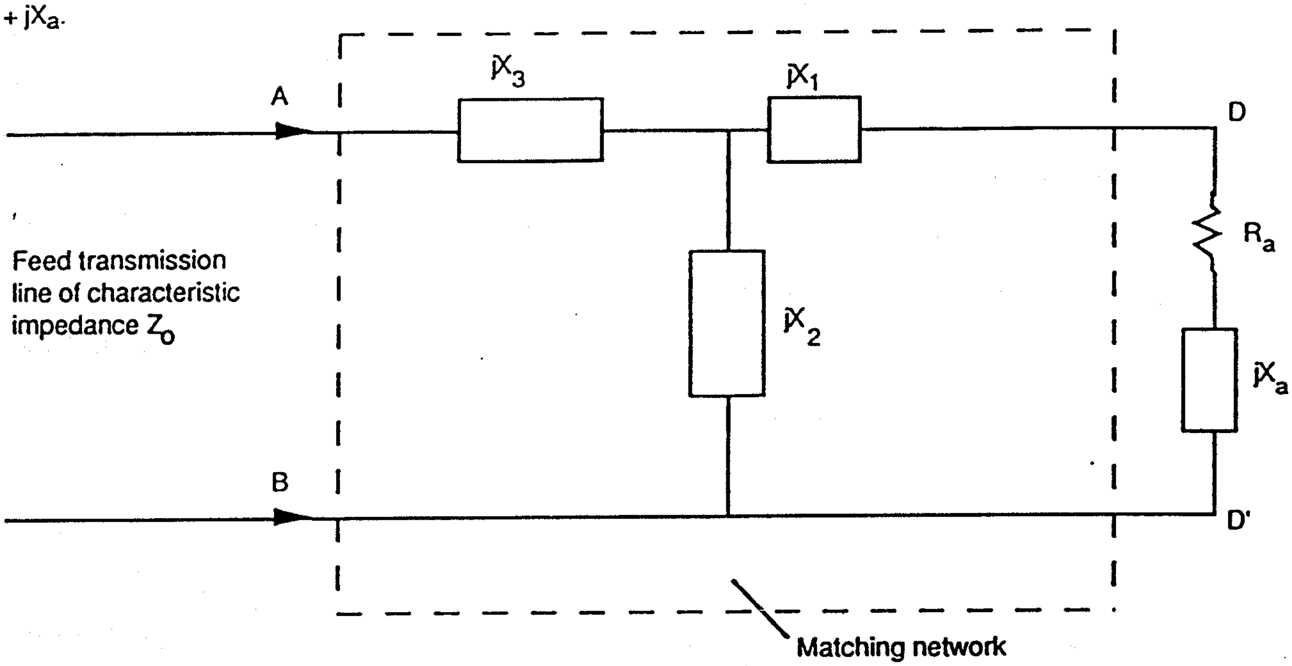

Fig. 1.

In order to match power into the antenna, it is necessary that the impedance of the network between points A and B be purely resistive and have the same value as Z_o, the characteristic impedance of the transmission line.

From Fig. 1 the expression for the impedance Z_{AB} can be written as:

$$Z_{AB} = \frac{(R_a + jX_a + jX_1)\, jX_2}{R + jX_a + jX_1 + jX_2} + jX_3 \qquad (1)$$

We select X_1 and X_2 such that the reactance in the denominator of the first term is zero, i.e.,

$$X_a + X_1 + X_2 \equiv 0 \qquad (2)$$

Equation 1 can then be rewritten as:

$$Z_{AB} = \frac{(R_a - jX_2)\,(jX_2)}{R_a} + jX_3 \qquad (3)$$

We select X_2 such that

$$\frac{X_2^2}{R_a} = Z_o \qquad (4)$$

and X_3 such that

$$X_3 = -X_2 \qquad (5)$$

This would then give

$$Z_{AB} = Z_o + j0$$

and the antenna would then be matched onto the transmission line.

To illustrate the procedure by a *numerical example* let us say that the antenna is a monopole and its impedance has been calculated and found to be 1.5 - j 460 Ω.

Let us take Z_o = 300 ohms (we must of course make sure that the diameter of the feeder line is not overly thick for the current carrying requirement). From Eq. 4

$$X_2 = \pm\sqrt{1.5 \times 300} = 21.2\Omega$$

The upper sign corresponds to an inductance L = 21.2/ω and the lower sign corresponds to a capacitance $C = \frac{1}{\omega \times 21.2}$. We can use either type.

Case 1: For Inductive element X_2

$$X_2 = 21.2\omega$$

From Eq. 2

$$X_1 = -X_2 - X_a = -21.2 + 460$$

$$= 438.8\Omega$$

From Eq. 5

$$X_3 = -21.2\,\Omega$$

one possible matching network is therefore shown in Fig. 2.

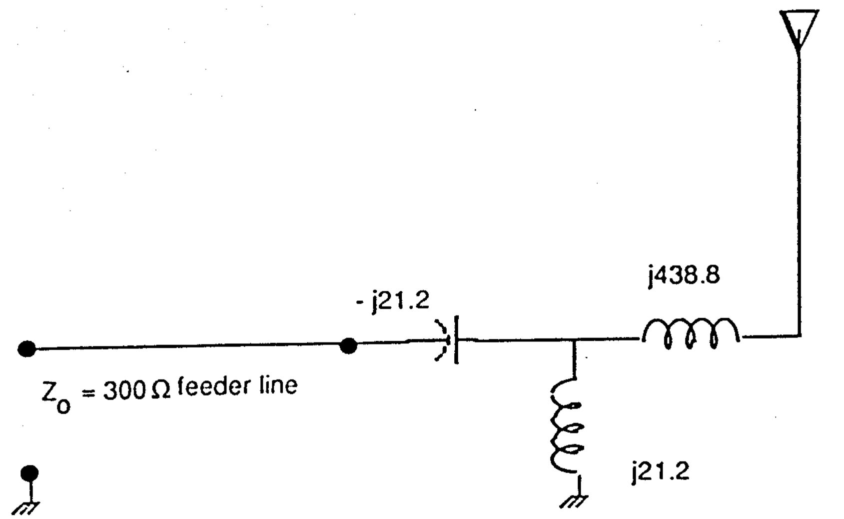

Fig. 2.

Case 2: For capacitive element X_2

$$X_2 = -21.2\Omega$$

From Eq. 2

$$X_1 = -X_2 - X_a = +21.2 + 460$$
$$= 481.2\Omega$$

From Eq. 5

$$X_3 = -X_2 = +21.2\Omega$$

and a second possible matching network is shown in Fig. 3.

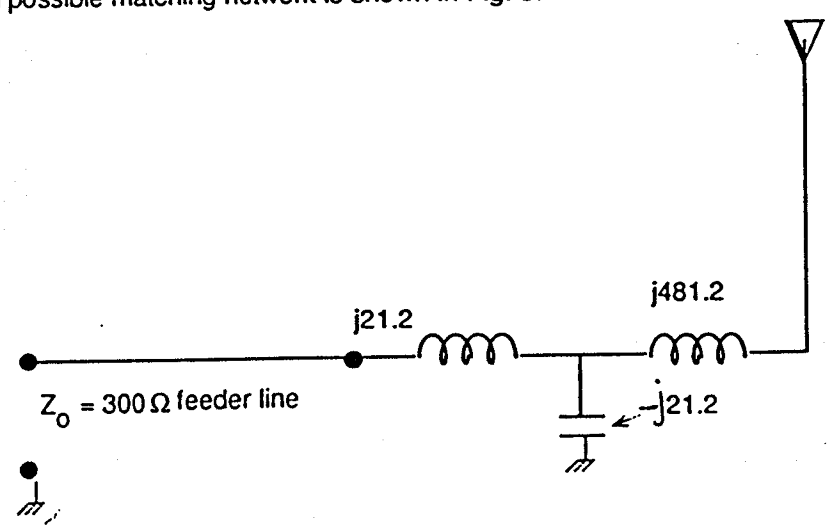

Fig. 3.

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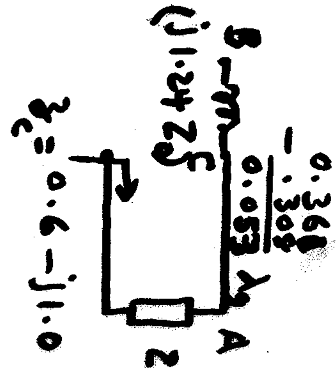

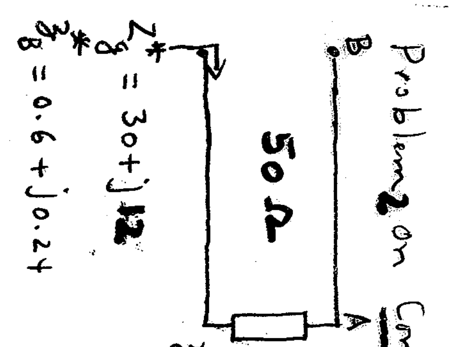

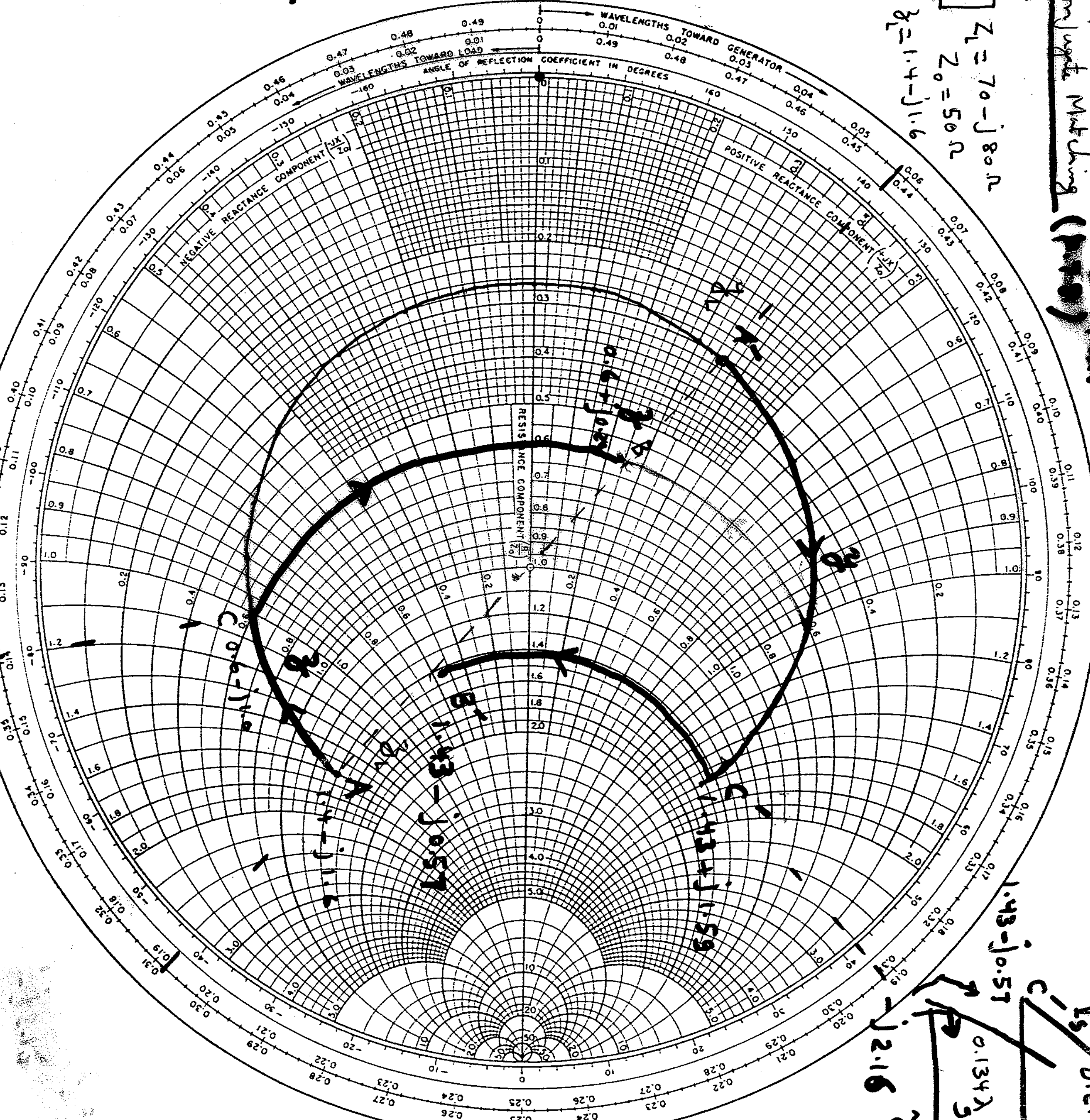

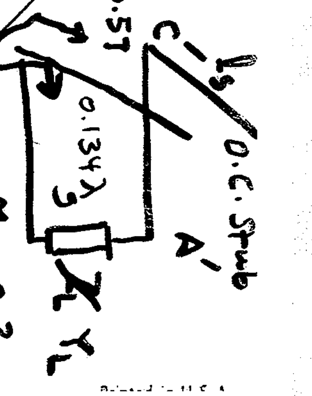

Ref: Phillip H. Smith "Transmission Line Calculator" Electronics Jan. 1939 and Jan. 1944

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