Notes on Chapter 4.

General Properties of Lossless Networks (see pp. 177, 178 - Text)

$$\begin{bmatrix} b_1 \\ b_2 \\ | \\ | \\ b_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & - - & S_{1N} \\ S_{21} & - & - - & S_{2N} \\ - & - & & - \\ S_{N1} & - & - - & S_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ a_i \\ \vdots \\ 0 \end{bmatrix}$$

$$b_1 = S_{1i}\, a_i$$
$$b_2 = S_{2i}\, a_i$$
$$- \quad - \quad -$$
$$b_N = S_{Ni}\, a_i$$

$$b_1 b_1^* + - - - b_N b_N^* = (S_{1i} S_{1i}^* + S_{2i} S_{2i}^* + - - S_{Ni} S_{Ni}^*)(a_i a_i^*)$$

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1 \quad \text{for any } i \quad i = 1, 2 - - - \qquad (4.53a)$$

(p. 178)

Property I: Sum of the squares of the S-parameters in any column = 1

II. Property 2 ~~Let us~~ feed power to any 2 ports say i, j

$$b_1 = S_{1i}\, a_i + S_{1j}\, a_j$$
$$- \quad - \quad - \quad - \quad -$$
$$b_N = S_{Ni}\, a_i + S_{Nj}\, a_j$$

$$b_1 b_1^* + - - b_N b_N^* = S_{1i} S_{1i}^* a_i a_i^* + S_{1j} S_{1j}^* a_j a_j^*$$
$$+ S_{1i} S_{1j}^* a_i a_j^* + S_{1i}^* S_{1j} a_i^* a_j + - - - = a_i a_i^* + a_j a_j^*$$

$$\left(\sum_{k=1}^{N} S_{ki} S_{ki}^*\right) a_i a_i^* + \left(\sum_{k=1}^{N} S_{kj} S_{kj}^*\right) a_j a_j^* + \left(\sum_{k=1}^{N} S_{ki} S_{kj}^*\right) a_i a_j^*$$
$$+ \left(\sum_{k=1}^{N} S_{ki}^* S_{kj}\right) a_i^* a_j = a_i a_i^* + a_j a_j^* \qquad (1)$$

Since a_i and a_j are arbitrary both for magnitude and phase, we can select them in phase

From Eq. (1) we can write
$$\sum_{k=1}^{N} S_{ki} S_{kj}^* + S_{ki}^* S_{kj} = 0$$

We can also select a_i, a_j^* to be 90° out of phase say $a_j^* = j\gamma a_i$ in which case we can write
$$\sum S_{ki} S_{kj}^* - S_{ki}^* S_{kj} = 0$$

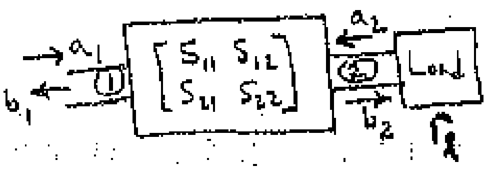
so $$\sum_{1}^{N} S_{ki} S_{kj}^* = \sum_{1}^{N} S_{ki}^* S_{kj} = 0 \qquad (4.53b) \quad \text{for } i \neq j$$

Ex.1 For a two-port circuit, calculate **(See also Ex. 4.5 Text p.179)**

a. the reflection coefficient Γ_1 at port ① if port ② is connected to a mismatched load of reflection Γ_l

b. the power delivered to this mismatched load in terms of input power $a_1 a_1^*$

Solution a. The circuit diagram is given in Fig. 1



$$b_1 = S_{11} a_1 + S_{12} a_2 \quad (1)$$
$$b_2 = S_{21} a_1 + S_{22} a_2 \quad (2)$$

For the mismatched load of reflection Γ_l, note that $\underline{a_2 = \Gamma_l b_2}$ (3)

Eq. (3) can be rewritten as
$$\boxed{b_2 = \frac{a_2}{\Gamma_l}} \quad (4)$$

Substituting Eq. (4) into Eq. (2) and rearranging terms
$$S_{21} a_1 = \left(\frac{1}{\Gamma_l} - S_{22}\right) a_2 = \left(\frac{1 - S_{22}\Gamma_l}{\Gamma_l}\right) a_2 \quad (5)$$

Substitute Eq. (5) into Eq. (1), we can write

Eq. 11.3a p. 537 3rd Ed., 12.3a p. 5[illegible] 4th Ed.

$$\boxed{\Gamma_1 = \frac{b_1}{a_1} = \left[S_{11} + \frac{S_{12} S_{21} \Gamma_l}{1 - S_{22}\Gamma_l}\right]} \quad (6)$$

b. Power delivered to the mismatched load $= b_2 b_2^* - a_2 a_2^* = b_2 b_2^* \left(1 - |\Gamma_l|^2\right)$ (7)

Expressing Eq. (2) in terms of a_1, we can write (by using Eq. (5))
$$b_2 = \left[S_{21} + \frac{S_{22} S_{21}\Gamma_l}{1 - S_{22}\Gamma_l}\right] a_1 = \left[\frac{S_{21}}{1 - S_{22}\Gamma_l}\right] a_1 \quad (8)$$

Combining Eqs. (7) and (8), we can write
$$\frac{\text{Power delivered to the mismatched load}}{\text{Power input } a_1 a_1^*} = \left|\frac{S_{21}}{1 - S_{22}\Gamma_l}\right|^2 \times \left(1 - |\Gamma_l|^2\right) \quad (9)$$

Ex.2 Calculate the quantities a. and b. in Ex. 1 for a circuit for which the S-parameters are $\begin{bmatrix} 0.1 & 0.8j \\ 0.8j & 0.2 \end{bmatrix}$ and $\underline{\Gamma_l = 0.333}$

Solution

a. From Eq. (6), $\frac{b_1}{a_1} = \Gamma_1 = \left[S_{11} + \frac{S_{12} S_{21}\Gamma_l}{1 - S_{22}\Gamma_l}\right] = -0.1285$

Return loss $RL = -20 \log\left|\frac{b_1}{a_1}\right| = 17.82\text{ dB}$

b. From Eq. (9), $\frac{\text{power delivered to the load}}{\text{input power } a_1 a_1^*} = \underline{0.654}$

i.e. 65.4 % of the input power can be delivered to this mismatched load

c. For a perfectly matched load connected to the output port ②, $\Gamma_l = 0$ and $\frac{\text{power delivered to this perfectly matched load}}{\text{input power}} = |S_{21}|^2 = (0.8)^2 = \underline{0.64}$

d. Max power delivered to the load for $\Gamma_l = S_{22}^*$ ⇒ 0.667 $a_1 a_1^*$

Stephen F. Adam

Microwave Theory and Applications

Prentice-Hall, Inc. 1969

3.2 FLOW GRAPH REPRESENTATION OF MICROWAVE NETWORKS

3.2.1 SCATTERING PARAMETERS

In analyzing microwave transmission line problems, one would like to find some generalized parameters to write for a network in question—parameters which can be measured with reasonable simplicity, even in microwave frequencies. Analysis of the energy flow through a two-port network is one way to do this.

A simple two-port network can be shown as a "black box" (Fig. 3.2-1).

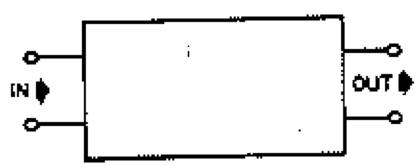


Fig. 3.2-1. Simple two-port network.

We are not interested in knowing what is built into the black box, but only in what it will do to a signal applied to either port. For example, if the black box contains an amplifier and we would like to know the various parameters, we can measure the input impedance while the output is short- and open-circuited and measure the output impedance while the input is short- and open-circuited. This will give us some of the commonly known z, y, and h parameters. However, this technique has some shortcomings at higher frequencies. Some devices may oscillate (probably at some frequency different from the measurement frequency) or have some unwanted, parasitic effects if they are terminated with a short or open circuit.

The ideal case would be to express a set of parameters when the input and output ports are terminated with their own characteristic impedances at all frequencies. The scattering (s) parameters are the set of parameters that are measured under such conditions. An added, inherent advantage of these

parameters is that they describe the signal flow within the network. Kurokawa,[1] Penfield,[2,3] and Youla[4] studied generalized scattering parameters. Hunton[5] used signal flow to analyze microwave-measurement techniques with s parameters and expressed them with flow graphs, since these parameters relate directly to the signal flow. Kuhn[6] used a topographical approach for resolving these flow graphs.

3.2.2 BASIC FLOW GRAPHS

A flow graph can be drawn to analyze the energy flow of a two-port network. (See Fig. 3.2-2.) A flow graph has two nodes for each port, one for

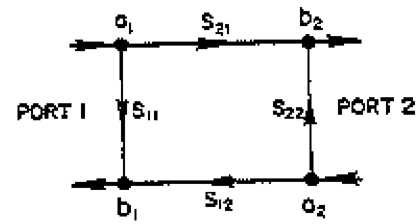


Fig. 3.2-2. Flow graph of a two-port network.

the entering (incident) wave and the other for the leaving (reflected) wave of that port. The incident node is the a node and the reflected node is the b node. In our example of the simple two-port network, when the incident wave enters the device at port 1, part of it will be returned through the s_{11} path and b_1 node. The remaining part of the incident wave goes through the s_{21} path and leaves the network through the b_2 node. If a device that has some reflections is connected to port 2, and if it will reflect part of the wave leaving b_2, this reflection will reenter the network through the a_2 node. Then, part of that may be reflected, passing along the s_{22} path and leaving the network

[1] Kurokawa, K., *IEEE Trans.-MTT*, March 1965, p. 194.

[2] Penfield, P., Jr., "Noise in Negative Resistance Amplifiers," *IRE Trans.-CT*, Vol. CT-7, June 1960, pp. 166–70.

[3] Penfield, P., Jr., "A Classification of Lossless Three Ports," *IRE Trans.-CT*, Vol. CT-9, September 1962, pp. 215–23.

[4] Youla, D. C., "On Scattering Matrices Normalized to Complex Port Numbers," *Proc. IRE*, Vol. 49, July 1961, p. 122.

[5] Hunton, J. K., "Analysis of Microwave Measurement Techniques by Means of Signal Flow Graphs," *Trans. IRE*, Vol. MTT-8, March 1960, pp. 206–12.

[6] Kuhn, Nicholaus, "Simplified Signal Flow Graph Analysis," *The Microwave Journal*, November 1963, pp. 59–66.

through the b_2 node. The other part of the wave passes through the s_{11} path and leaves the circuit through the b_1 node.

Following the arrows in the flow graph, we can write the following equations.

$$b_1 = a_1 s_{11} + a_2 s_{12} \tag{3.2-1}$$

$$b_2 = a_1 s_{21} + a_2 s_{22} \tag{3.2-2}$$

By analyzing these equations, one can see that these parameters can be easily measured under certain conditions.

Assume that there is no signal entering at a_2 node (this can be achieved by terminating port 2 with its characteristic impedance). Then Eqs. (3.2-1) and (3.2-2) become

$$b_1 = a_1 s_{11}, \tag{3.2-3}$$

$$b_2 = a_1 s_{21}. \tag{3.2-4}$$

By reversing the network, that is, terminating port 1 with its characteristic impedance and applying the signal to port 2, Eqs. (3.2-1) and (3.2-2) will become

$$b_1 = a_2 s_{12}, \tag{3.2-5}$$

$$b_2 = a_2 s_{22}, \tag{3.2-6}$$

since $a_1 = 0$.

Expressing the scattering parameters from Eqs. (3.2-3), (3.2-4), (3.2-5), and (3.2-6), we can write the following:

$$s_{11} = \frac{b_1}{a_1} \Big| \; a_2 = 0 \tag{3.2-7}$$

$$s_{21} = \frac{b_2}{a_1} \Big| \; a_2 = 0 \tag{3.2-8}$$

$$s_{12} = \frac{b_1}{a_2} \Big| \; a_1 = 0 \tag{3.2-9}$$

$$s_{22} = \frac{b_2}{a_2} \Big| \; a_1 = 0 \tag{3.2-10}$$

Furthermore, these expressions show the means of measuring these parameters: s_{11} can be measured when port 2 is terminated with its characteristic impedance and only the ratio of reflected wave and incident wave has to be measured at port 1. We saw that in Chap. 2, where reflection coefficients were discussed. This means that s_{11} is really the input-reflection coefficient of the device.

s_{22} is measured in exactly the same manner as s_{11}, except that port 1 is terminated with its characteristic impedance and the signal is applied to port 2; s_{22} is the output-reflection coefficient of the network.

s_{21} is measured when port 2 is terminated with its characteristic impedance and the signal is applied into port 1. The ratio of the signals measured at the b_2 and a_1 nodes (voltage between output and input ports) defines the value of s_{21}. Simply, s_{21} is the forward transducer coefficient.

s_{12} is measured by reversing the ports and terminating port 1 in its characteristic impedance and applying the signal to port 2. The ratio of the signals appearing at the b_1 and a_2 nodes will define the value of the s_{12} parameter. s_{12} is the reverse transducer coefficient of the network.

These parameters are vector values, and they have both magnitude and phase information.

It is much easier to make swept-frequency, wideband measurements of s parameters than of h, y, and z parameters, especially above 100 MHz. To use the many design techniques defined in terms of h, y, and z parameters, it is quite simple to convert data to any of these parameters from the scattering parameters. Table (3.2-1) shows the conversion equations for each of these parameters and the scattering parameters.

Table 3.2-1. Conversion Equations Between h, z, y, and s Parameters

$$s_{11} = \frac{(z_{11}-1)(z_{22}+1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \qquad z_{11} = \frac{(1+s_{11})(1-s_{22}) + s_{12}s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$$

$$s_{12} = \frac{2z_{12}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \qquad z_{12} = \frac{2s_{12}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$$

$$s_{21} = \frac{2z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \qquad z_{21} = \frac{2s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$$

$$s_{22} = \frac{(z_{11}+1)(z_{22}-1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \qquad z_{22} = \frac{(1+s_{22})(1-s_{11}) + s_{12}s_{21}}{(1-s_{11})(1-s_{22}) - s_{12}s_{21}}$$

$$s_{11} = \frac{(1-y_{11})(1+y_{22}) - y_{12}y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \qquad y_{11} = \frac{(1+s_{22})(1-s_{11}) + s_{12}s_{21}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}$$

$$s_{12} = \frac{-2y_{12}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \qquad y_{12} = \frac{-2s_{12}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}$$

$$s_{21} = \frac{-2y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \qquad y_{21} = \frac{-2s_{21}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}$$

$$s_{22} = \frac{(1+y_{11})(1-y_{22}) + y_{12}y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \qquad y_{22} = \frac{(1+s_{11})(1-s_{22}) + s_{12}s_{21}}{(1+s_{22})(1+s_{11}) - s_{12}s_{21}}$$

$$s_{11} = \frac{(h_{11}-1)(h_{22}+1) - h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \qquad h_{11} = \frac{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

$$s_{12} = \frac{2h_{12}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \qquad h_{12} = \frac{2s_{12}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

$$s_{21} = \frac{-2h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \qquad h_{21} = \frac{-2s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

$$s_{22} = \frac{(1+h_{11})(1-h_{22}) + h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \qquad h_{22} = \frac{(1-s_{22})(1-s_{11}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

3.2.3 TOPOGRAPHICAL APPROACH TO RESOLVE FLOW GRAPHS[7]

It was emphasized in the previous section that the scattering parameters are descriptive of signal flow; consequently, signal flow graphs can easily show the scattering parameters as signal flow elements. A two-port network has been described already. The flow graph of a three-port network can be realized in the same manner. Figure 3.2-3 shows such a flow graph.

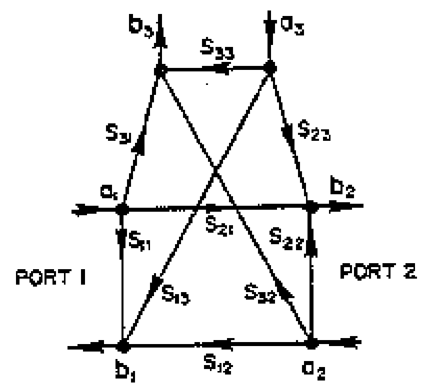


Fig. 3.2-3. Flow graph of a three-port network.

Nodes that represent waves entering and leaving the network are designated a_n and b_n, respectively. There is always a connecting line from an a_n node to a b_n node within the network flow graph, and these connecting lines always go from a to b. They are associated with an s parameter.

Networks can be cascaded one after the other, and their flow graphs can be cascaded similarly, as in Fig. 3.2-4, which shows two two-port networks treated in this way. It is interesting to note that node b_2 and a_1' are synonymous; a_2 and b_1' are also synonymous. In a flow graph, synonymous nodes can be connected with an arrow having a value of "1," meaning that there is no electrical length between them. These two groups of nodes should not be considered identical; the direction of the arrow between b_2 and a_1' is important. Basic transmission line elements can be divided into one-port, two-port, and multiport groups. Every port will have two nodes: one where the wave enters (a) and the other where the wave leaves that port (b).

Flow graph representation of some one-port networks is shown in Fig.

[7] Kuhn, "Simplified Signal Flow Graph Analysis," *Microwave Journal*, November 1963.

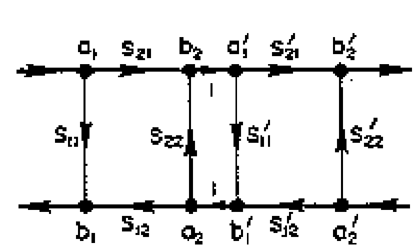


Fig. 3.2-4. Two two-port networks cascaded.

3.2-5. M is the meter reading of an indicator, as shown; K represents the law of the detector and does not change with power level so long as the detector law does not change with power level. Furthermore, M includes the effect of the transmission loss due to the detector's reflection $\sqrt{1-\rho_D^2}$.

Flow graphs of some two-port networks are shown in Fig. 3.2-6. These flow graphs are only the most-used elements. Remember that Γ stands for

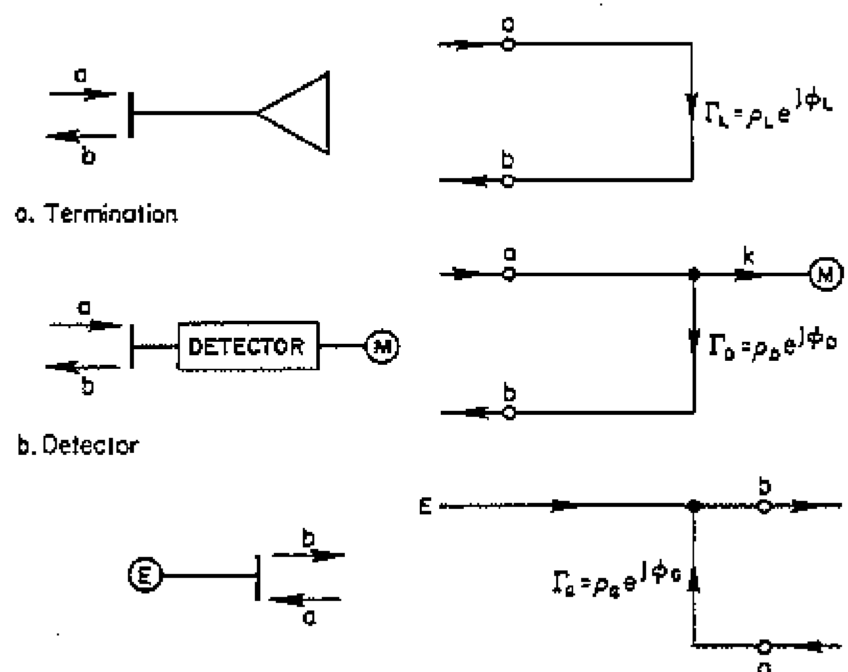


Fig. 3.2-5. Flow graph representation of some one-port networks.

Figure 3.2-18 shows the flow graph of a two-port network driven with a signal source and terminated with a load. One path goes from the generator to node b_2; its value is s_{21}. There are two paths from the generator to node b_1. The values of these paths are s_{11} and $s_{21}\Gamma_L s_{12}$.

If a path starts and finishes in the same node, it is called a "loop," rather than a path. A "first-order loop" is a path coming to a closure with no node passed more than once. The value of the loop is calculated as the value of the path, or the product of the value of all branches encountered en route.

A "second-order loop" is defined as two first-order loops not touching each other at any node. The value of a second-order loop is the product of

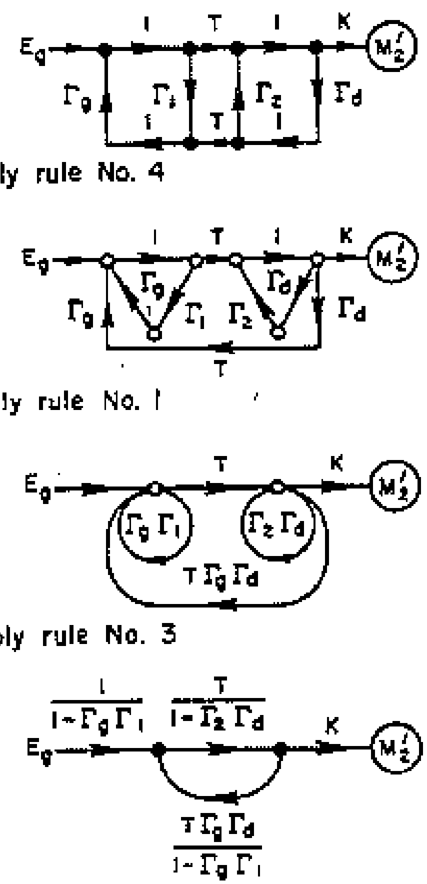


Fig. 3.2-17. Reduction of the measurement flow graph.

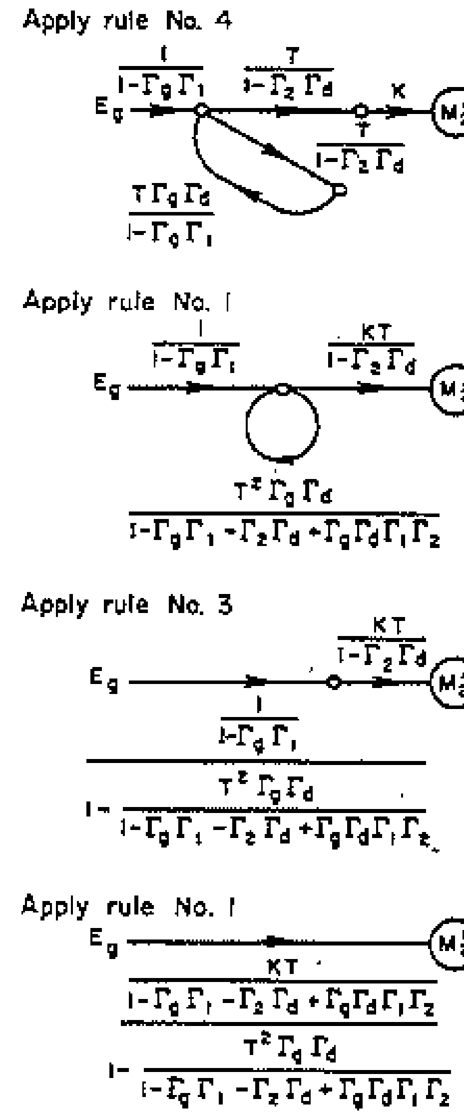


Fig. 3.2-17 (*continued*)

the values of the two first-order loops. Third- and higher-order loops are three or more first-order loops not touching each other at any point. Their values are calculated in the same manner as described above for the second-order loop, that is, by multiplying the coefficients of branches encountered.

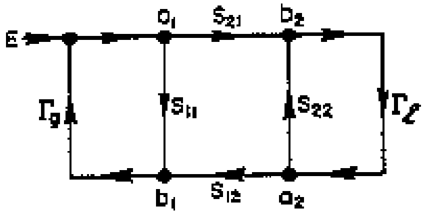


Fig. 3.2-18. Flow graph of a two-port network with a signal source and a load.

For example, in Fig. 3.2-18, there are three first-order loops ($s_{11}\Gamma_g$, $s_{22}\Gamma_L$, and $\Gamma_g s_{21}\Gamma_L s_{12}$) and one second-order loop ($\Gamma_g s_{11} s_{22}\Gamma_L$).

The nontouching-loop rule[6,7] can be applied to solve any flow graph. The equation in symbolic form is

$$T = \frac{P_1[1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \Sigma L(3)^{(1)} + \ldots] + P_2[1 - \Sigma L(1)^{(2)} + \Sigma L(2)^{(2)} - \ldots] + P_3[1 - \Sigma L(1)^{(3)} + \ldots] + P_4(1 - \ldots) +}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \ldots}$$

where $\Sigma L(1)$ stands for the sum of all first-order loops, $\Sigma L(2)$ is the sum of all second-order loops, and so on; P_1, P_2, P_3, etc., stand for the values of all paths that can be followed from the independent variable, in most cases the generator, to the node whose value is desired; $\Sigma L(1)^{(1)}$ denotes the sum of those first-order loops which do not touch the path of P_1 at any node; $\Sigma L(2)^{(1)}$ denotes then the sum of those second-order loops which do not touch the path of P_1 at any point; $\Sigma L(1)^{(2)}$ consequently denotes the sum of those first-order loops which do not touch the path of P_2 at any point. Each path is multiplied by the factor in parentheses which involves all the loops of all orders that the path does not touch. T represents the ratio of the dependent variable in question and the independent variable.

The example shown in Fig. 3.2-18 can be calculated for two dependent variables. One is the reflection coefficient of the two-port network b_1/a_1, and the second is the transmission coefficient b_2/E. In the first case, when b_1/a_1 is to be found, the generator is not involved, so it should be neglected. The solution is

$$\frac{b_1}{a_1} = \frac{s_{11}(1 - s_{22}\Gamma_L) + s_{21}\Gamma_L s_{12}}{1 - s_{22}\Gamma_L}$$

s_{11} is the first path, P_1, which has to be multiplied with $1 - \Sigma L(1)^{(1)}$. $s_{22}\Gamma_L$ is

[6] Lorens, C. S., "A Proof of the Nonintersecting Loop Rule for the Solution of Linear Equations by Flow Graphs," Res. Lab. of Electronics, M.I.T., Cambridge, Mass. *Quarterly Progress Report*, January 1956, pp. 97–102.

[7] Happ, W. W., "Lecture Notes on Signal Flow Graphs," from *Analysis of Transistor Circuits*, Extension Course, University of California, Catalog 834AB.

the only first-order loop not touching the P_1 path; higher-order loops not touching the P_1 path do not exist. Path number two, P_2, will be $s_{21}\Gamma_L s_{12}$; since there are no first-order or any higher-order loops not touching this path, it will be multiplied by 1. The denominator shows the only first-order loop, $s_{22}\Gamma_L$, subtracted from unity.

The entire flow graph, including the generator, is needed to write the solution for the transmission coefficient.

$$\frac{b_2}{E} = \frac{s_{21}}{1 - \Gamma_g s_{11} - s_{22}\Gamma_L - \Gamma_g s_{21}\Gamma_L s_{12} + \Gamma_g s_{11} s_{22}\Gamma_L}$$

Because there is only one possible path from E to the b_2 node, and there are no loops not touching this path, only s_{21} will stay in the numerator. It can be seen that there are three first-order loops and a second-order loop in the denominator.

It would be interesting to see the attenuation measurement flow graph discussed in the topographical approach as another example. Figure 3.2-15 shows the flow graphs in question. Equations have to be written for M_1'/E_g and M_2'/E_g; the values of M_1/E_g and M_2/E_g have already been found analytically.

$$\frac{M_1'}{E_g} = \frac{k}{1 - \Gamma_g\Gamma_d}$$

since k is the only path and $\Gamma_g\Gamma_d$ is the only loop.

$$\frac{M_2'}{E_g} = \frac{kT}{1 - \Gamma_g\Gamma_1 - \Gamma_2\Gamma_d - \Gamma_g T^2\Gamma_d + \Gamma_g\Gamma_1\Gamma_2\Gamma_d}$$

Again the only path is kT, and all loops touch this path. Three first-order loops and a second-order loop can be found in the denominator.

It is worth mentioning that third- and higher-order loops can usually be neglected after careful analysis of the values of various coefficients in question. This is because values smaller than unity multiplied with each other become even smaller. This point will be emphasized later in the text.

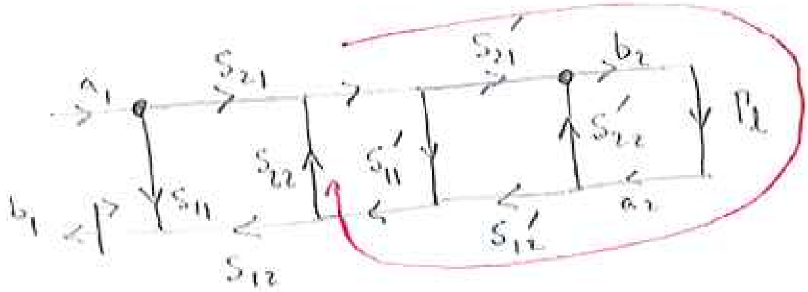
Nontouching-Loop Rule

$$T = \frac{P_1(1 - \Sigma L(1)^{(1)} + \Sigma L(2)^{(1)} - \Sigma L(3)^{(1)} + \ldots) + P_2(1 - \Sigma L(1)^{(2)} + \Sigma L(2)^{(2)} - \ldots) + P_3(1 - \Sigma L(1)^{(3)} + \ldots) + \ldots}{1 - \Sigma L(1) + \Sigma L(2) - \Sigma L(3) + \ldots}$$

$\Sigma L(1)$	Sum of all first-order loops
$\Sigma L(2)$	Sum of all second-order loops
P_1, P_2, P_3	Values of paths corresponding to indices
$\Sigma L(1)^{(1)}$	Sum of those first-order loops which do not touch P_1
$\Sigma L(2)^{(1)}$	Sum of those second-order loops which do not touch P_1
$\Sigma L(n)^{(m)}$	Sum of those n-order loops which do not touch P_m path
T	Ratio of dependent variable in question and independent variable

Similar to Prob. 4.11 Text p. 271

Two 2-ports in cascade (see p. 91 of the handout) e.g. a 2-stage amplifier



calculate $\frac{b_1}{a_1}$

Paths between b_1 and a_1

	Value	First order Nontouching Loops $L(1)^{(1)}$	2nd order Nontouching Loop $L(2)^{(1)}$
P_1	S_{11}	$S'_{11}S_{22}$, $\Gamma_l S'_{22}$, $S_{22}S'_{21}\Gamma_l S'_{12}$	$(S'_{11}S_{22})(\Gamma_l S'_{22})$
P_2	$S_{21}S'_{11}S_{12}$	$\Gamma_l S'_{22}$	0
P_3	$S_{21}S'_{21}\Gamma_l S'_{12}S_{12}$	—	—

$$\Gamma_1 = \frac{b_1}{a_1} = S_{11} + \frac{S_{21}S'_{11}S_{12}\left[1-\Gamma_l S'_{22}\right] + S_{21}S'_{21}\Gamma_l S'_{12}S_{12}}{1-\left\{S'_{11}S_{22}+\Gamma_l S'_{22}+S_{22}S'_{21}\Gamma_l S'_{12}\right\}+\left(S'_{11}S_{22}\right)\left(\Gamma_l S'_{22}\right)}$$

← Second order loop

$S_{11}\big|_{\text{load circuit}}$ ... $\Gamma_1\big|_{a_2=0} \Rightarrow \Gamma_1\big|_{\Gamma_l=0} = S_{11} + \frac{S_{21}S'_{11}S_{12}}{1-S'_{11}S_{22}}$

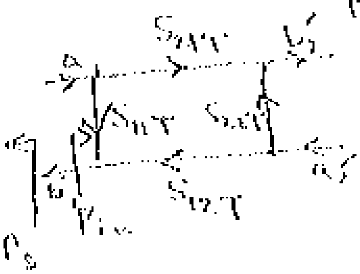
$$\frac{b_2}{a_1} = \frac{S_{21}S'_{21}\left[1-0\right]}{1-\left(S'_{22}\Gamma_l + S'_{11}S_{22} + S'_{21}\Gamma_l S'_{12}S_{22}\right) + \left(S'_{11}S_{22}\right)\left(\Gamma_l S'_{22}\right)}$$

$$S_{21T} \rightarrow \frac{b_2}{a_1}\bigg|_{\text{if } \Gamma_L=0} \Rightarrow \frac{S_{21}S'_{21}}{1-S'_{11}S_{22}} \Rightarrow \frac{S^A_{21}\,S^B_{21}}{1-S^B_{11}\,S^A_{22}}$$

Power delivered to the load (P_L) = $b_2 b_2^*\left(1-|\Gamma_l|^2\right)$

Fractional power reflected = $\frac{b_1 b_1^*}{a_1 a_1^*}$

Two 2-ports in cascade



$$S_{12T} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{S_{12}\, S_{12}'}{1 - S_{11}'\, S_{22}}$$

$$S_{22T} = \left.\frac{b_2}{a_2}\right|_{\substack{a_1=0 \\ \Gamma_{in}=0}} = S_{22}' + \frac{S_{12}'\, S_{22}\, S_{21}'}{1 - S_{11}'\, S_{22}} \quad \text{when } \Gamma_S = 0$$

Incorporating the S_{11T} and S_{21T} for the two 2-port circuits in cascade, the equivalent S-parameters of the combined 2-port circuit can be written as

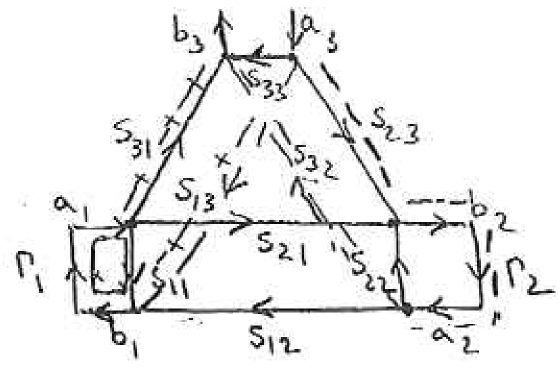
$$[S]_T = \begin{bmatrix} S_{11T} & S_{12T} \\ S_{21T} & S_{22T} \end{bmatrix}$$

$$= \begin{bmatrix} S_{11} + \dfrac{S_{21} S_{11}' S_{12}}{1 - S_{11}' S_{22}} & \dfrac{S_{12} S_{12}'}{1 - S_{11}' S_{22}} \\ \dfrac{S_{21} S_{21}'}{1 - S_{11}' S_{22}} & S_{22}' + \dfrac{S_{12}' S_{22} S_{21}'}{1 - S_{11}' S_{22}} \end{bmatrix}$$

A handicap of the S-parameters is that for cascaded circuits, there is no easy way to combine the S-parameters of the individual circuits. This handicap is avoided by introducing A, B, C, D parameters in Section 4.4 of the Text (which are not easy to measure (because of the difficulty of measuring voltages and currents) but do allow multiplication of A B C D matrices of the circuits in cascade.

Example of a 3 port network (see also p. 90 of the handout from Adam's book)

Find the reflection coefficient at port 3 if ports 1 and 2 are mismatched with reflection coefficients Γ_1 and Γ_2, respectively.



Do not draw any branches for which the value is zero

Paths between a_3 and b_3

		Value	First order Nontouching loops $L(1)^{(1)}$	Second order Nontouching loops $L(2)^{(1)}$
═	P_1	S_{33}	$\Gamma_1 S_{11}$, $\Gamma_2 S_{22}$, $\Gamma_1 S_{21} \Gamma_2 S_{12}$	$(\Gamma_1 S_{11})(\Gamma_2 S_{22})$
– – –	P_2	$S_{23}\Gamma_2 S_{32}$	$\Gamma_1 S_{11}$	—
–x–	P_3	$S_{13}\Gamma_1 S_{31}$	$\Gamma_2 S_{22}$	—
	P_4	$S_{23}\Gamma_2 S_{12}\Gamma_1 S_{31}$	—	—
	P_5	$S_{13}\Gamma_1 S_{21}\Gamma_2 S_{32}$		

All First order loops touching or nontouching $L(1)$

$\Gamma_1 S_{11}$, $\Gamma_2 S_{22}$, $\Gamma_1 S_{21}\Gamma_2 S_{12}$

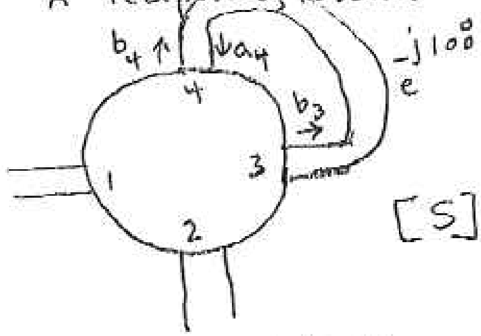
All second order loops touching or nontouching $L(2)$

$(\Gamma_1 S_{11})(\Gamma_2 S_{22})$

$$\frac{b_3}{a_3} = S_{33} + \frac{S_{23}\Gamma_2 S_{32}(1-\Gamma_1 S_{11}) + S_{13}\Gamma_1 S_{31}(1-\Gamma_2 S_{22}) + S_{23}\Gamma_2 S_{12}\Gamma_1 S_{31} + S_{13}\Gamma_1 S_{21}\Gamma_2 S_{32}}{1-(\Gamma_1 S_{11}+\Gamma_2 S_{22}+\Gamma_1 S_{21}\Gamma_2 S_{12}) + (\Gamma_1 S_{11})(\Gamma_2 S_{22})}$$

similar to Prob 4.17 8 Text

A reciprocal, lossless 4-port network

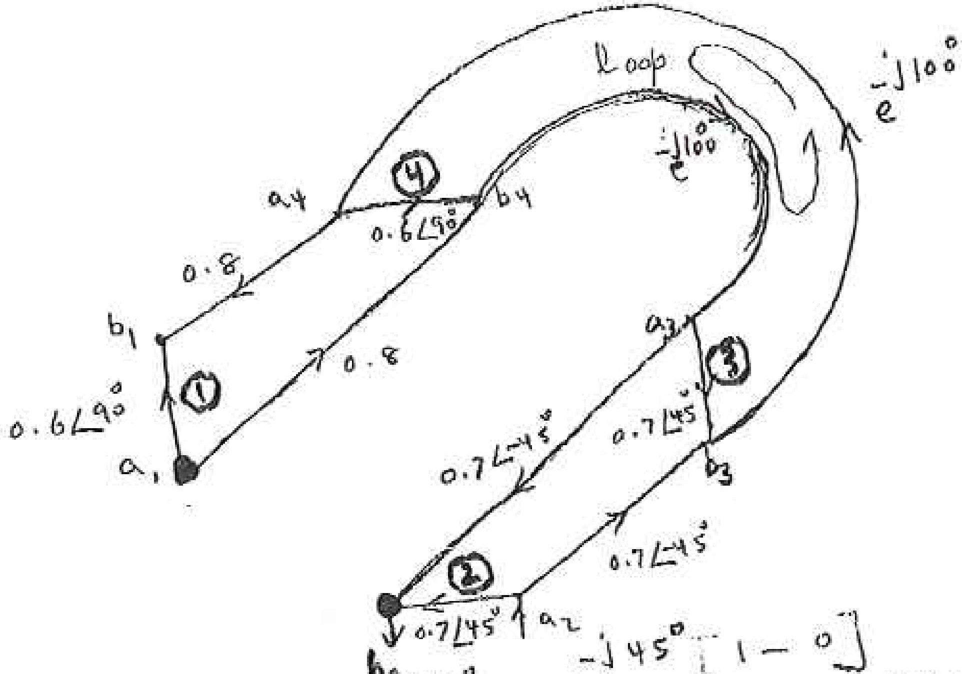


$a_4 = b_3\, e^{-j100°}$

$a_3 = b_4\, e^{-j100°}$

$$[S] = \begin{bmatrix} 0.6\angle 90° & 0 & 0 & 0.8\angle 0 \\ 0 & 0.7\angle 45° & 0.7\angle -45° & 0 \\ 0 & 0.7\angle -45° & 0.7\angle 45° & 0 \\ 0.8\angle 0° & 0 & 0 & 0.6\angle 90° \end{bmatrix}$$

Note that ports 3 and 4 are connected with a lossless matched transmission line of electrical length 100°



$$\frac{b_2}{a_1} = \frac{\text{path } P_1 = 0.8 \times e^{-j100°} \times 0.7\, e^{-j45°}\,[1 - 0]}{1 - \left[(0.6\, e^{+j90°})(e^{-j100°})(0.7\, e^{j45°})(e^{-j100°})\right]}$$

$$= \frac{0.56\, e^{-j145°}}{1 - 0.42\, e^{-j65°}}$$

$$\frac{b_2 b_2^*}{a_1 a_1^*} = \frac{(0.56)^2}{(1 - 0.42\, e^{-j65°})(1 - 0.42\, e^{+j65°})} = \frac{(0.56)^2}{1 + (0.42)^2 - 2 \times 0.42 \cos 65°}$$

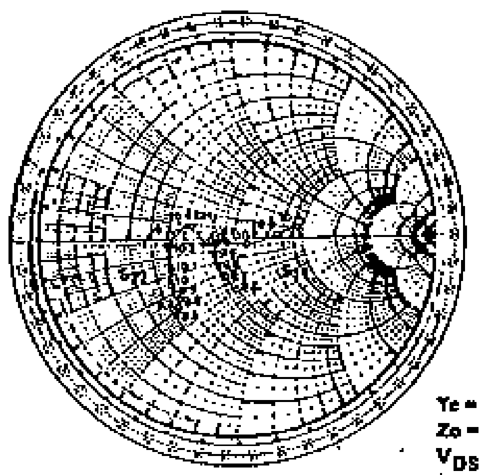
$$= \boxed{0.4648}$$

$IL = -10 \log(0.4648) = -3.33\ \text{dB}$

P_o = 2.3 W (Typ.) @ 10 GHz
G_p = 5 dB (Typ.) @ 10 GHz

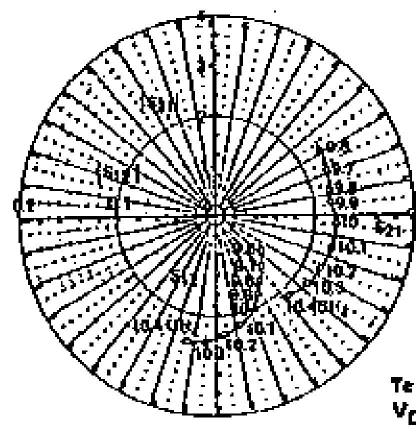
MITSUBISHI GaAs POWER FET
MGF-X34M
INTERNALLY MATCHING, FLIP-CHIP MOUNTING

S_{11}, S_{22} vs. f



Tc = 25°C
Zo = 50 Ω
V_{DS} = 9 V
I_D = 900 mA

S_{12}, S_{21} vs. f



Tc = 25°C
V_{DS} = 9 V
I_D = 900 mA

S PARAMETERS (Tc = 25°C, V_{DS} = 9 V, I_D = 900 mA)

f (GHz)	S Parameters (TYP.)							
	S_{11}		S_{12}		S_{21}		S_{22}	
	Magn.	Angle (deg.)	Magn.	Angle (deg.)	Magn.	Angle (deg.)	Magn.	Angle (deg.)
9.6	0.253	−90.0	0.058	−39.4	2.51	31.2	0.470	−129.8
9.7	0.193	−105.5	0.068	−48.2	2.44	21.9	0.447	−132.8
9.8	0.154	−124.8	0.077	−55.4	2.37	13.1	0.409	−132.9
9.9	0.127	−143.6	0.087	−60.7	2.37	5.6	0.410	−136.5
10.0	0.097	−165.9	0.100	−68.4	2.43	−4.5	0.402	−145.4
10.1	0.036	−156.3	0.111	−78.0	2.42	−15.4	0.359	−155.5
10.2	0.068	−20.2	0.118	−85.8	2.36	−26.5	0.333	−171.7
10.3	0.203	3.5	0.126	−95.2	2.30	−37.5	0.301	173.3
10.4	0.309	10.3	0.128	−103.4	2.17	−48.2	0.227	164.7

Self reflection coeff. | Feedback coeff. | forward gain | Self refl. coeff.

FHX04FA/LG Fujitsu HEMT (89/90), f=0 extrapolated; Vds=2V, Ids=10mA

f	s11		s21		s12		s22	
0.0	1.000	0.0	4.375	180.0	0.000	0.0	0.625	0.0
1.0	0.982	-20.0	4.257	160.4	0.018	74.8	0.620	-15.2
2.0	0.952	-39.0	4.113	142.0	0.033	62.9	0.604	-28.9
3.0	0.910	-57.3	3.934	124.3	0.046	51.5	0.585	-42.4
4.0	0.863	-75.2	3.735	107.0	0.057	40.3	0.564	-55.8
5.0	0.809	-92.3	3.487	90.4	0.065	30.3	0.541	-69.2
6.0	0.760	-108.1	3.231	75.0	0.069	21.0	0.524	-82.0
7.0	0.727	-122.4	3.018	60.9	0.072	14.1	0.521	-93.6
8.0	0.701	-135.5	2.817	47.3	0.073	7.9	0.524	-104.7
9.0	0.678	-147.9	2.656	33.8	0.074	1.6	0.538	-115.4
10.0	0.653	-159.8	2.512	20.2	0.076	-4.0	0.552	-125.7
11.0	0.623	-171.1	2.367	7.1	0.076	-10.1	0.568	-136.4
12.0	0.601	178.5	2.245	-5.7	0.076	-15.9	0.587	-146.4
13.0	0.582	168.8	2.153	-18.4	0.076	-21.9	0.611	-156.2
14.0	0.564	160.2	2.065	-31.2	0.077	-28.6	0.644	-165.4
15.0	0.533	151.6	2.001	-44.5	0.079	-36.8	0.676	-174.8
16.0	0.500	142.8	1.938	-58.8	0.082	-48.5	0.707	174.2
17.0	0.461	134.3	1.884	-73.7	0.083	-61.7	0.733	163.6
18.0	0.424	126.6	1.817	-89.7	0.085	-77.9	0.758	150.9
19.0	0.385	121.7	1.708	-106.5	0.087	-97.2	0.783	139.1
20.0	0.347	119.9	1.613	-123.7	0.098	-119.9	0.793	126.6

ATF-21186 Typical Scattering Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 2\ V$, $I_{DS} = 15\ mA$

Frequency MHz	S11 Mag.	S11 Ang.	S21 Mag.	S21 Ang.	S12 Mag.	S12 Ang.	S22 Mag.	S22 Ang.
500	0.98	-49	3.77	147	0.069	62	0.34	-55
1000	0.92	-51	3.42	133	0.092	54	0.33	-63
2000	0.81	-87	2.85	108	0.131	39	0.32	-81
3000	0.72	-114	2.41	84	0.159	25	0.29	-105
4000	0.64	-143	2.11	61	0.178	13	0.26	-135
5000	0.62	-172	1.83	39	0.186	1	0.25	-170
6000	0.61	162	1.59	19	0.189	-9	0.28	162
7000	0.63	140	1.39	2	0.192	-15	0.33	144
8000	0.65	123	1.25	-14	0.200	-20	0.37	129

ATF-21186 Typical Scattering Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 2\ V$, $I_{DS} = 20\ mA$

Frequency MHz	S11 Mag.	S11 Ang.	S21 Mag.	S21 Ang.	S12 Mag.	S12 Ang.	S22 Mag.	S22 Ang.
500	0.98	-51	4.17	146	0.066	61	0.31	-62
1000	0.91	-64	3.73	132	0.087	54	0.30	-69
2000	0.80	-90	3.11	107	0.123	40	0.28	-87
3000	0.70	-118	2.60	83	0.150	27	0.26	-111
4000	0.63	-147	2.25	60	0.169	16	0.23	-142
5000	0.61	-176	1.94	39	0.180	5	0.23	-175
6000	0.61	159	1.68	20	0.187	-4	0.28	155
7000	0.62	137	1.47	2	0.193	-11	0.33	138
8000	0.64	120	1.32	-13	0.205	-17	0.36	125

ATF-21186 Typical Scattering Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 3\ V$, $I_{DS} = 70\ mA$

Frequency MHz	S11 Mag.	S11 Ang.	S21 Mag.	S21 Ang.	S12 Mag.	S12 Ang.	S22 Mag.	S22 Ang.
500	0.95	-30	6.69	156	0.04	74	0.31	-26
1000	0.87	-57	5.74	136	0.06	63	0.20	-58
2000	0.69	-105	4.30	102	0.10	48	0.13	-109
3000	0.63	-139	3.36	80	0.12	41	0.18	-149
4000	0.62	-170	2.63	60	0.14	33	0.22	-178
5000	0.63	165	2.12	42	0.15	28	0.27	161
6000	0.67	147	1.76	25	0.17	23	0.33	146
7000	0.71	133	1.49	13	0.18	18	0.39	136
8000	0.74	125	1.29	5	0.19	16	0.43	131

ATF-21186 Typical Noise Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 2\ V$, $I_{DS} = 10\ mA$

Frequency	NF_o	Γ_{opt}		RN/50 Ω
MHz	dB	Mag.	Ang.	-
500	0.57	0.83	30	0.78
1000	0.61	0.80	39	0.58
2000	0.71	0.76	61	0.48
4000	0.90	0.72	109	0.23
6000	1.09	0.71	168	0.04
8000	1.29	0.73	-145	0.10

ATF-21186 Typical Noise Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 2\ V$, $I_{DS} = 15\ mA$

Frequency	NF_o	Γ_{opt}		RN/50 Ω
MHz	dB	Mag.	Ang.	-
500	0.50	0.91	31	0.69
1000	0.55	0.87	40	0.49
2000	0.65	0.77	63	0.40
4000	0.84	0.66	111	0.29
6000	1.13	0.65	171	0.04
8000	1.25	0.79	-141	0.11

ATF-21186 Typical Noise Parameters,

Common Source, $Zo = 50\ \Omega$, $V_{DS} = 2\ V$, $I_{DS} = 20\ mA$

Frequency	NF_o	Γ_{opt}		RN/50 Ω
MHz	dB	Mag.	Ang.	-
500	0.45	0.88	31	0.56
1000	0.50	0.84	41	0.43
2000	0.60	0.75	64	0.35
4000	0.82	0.65	113	0.16
6000	1.03	0.65	174	0.03
8000	1.24	0.69	-139	0.11

pp. 537, 538 Text

Power Gain of a microwave Amplifier

We have previously derived (for an imperfectly matched output)

$$\Gamma_1 = \Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} \quad (11.3a)$$

This equation can be used to obtain Z_{in} since $\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$

We can similarly write Γ_2 (for an imperfectly matched input)

$$\Gamma_2 = \Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S} \quad (11.3b)$$

By voltage division

$$V_1 = V_{in} = V_S \frac{Z_{in}}{Z_{in} + Z_S} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in}) \quad (1)$$

Using $Z_{in} = Z_0 \dfrac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$ (2)

and $\Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0}$ which gives $Z_S = Z_0 \dfrac{1 + \Gamma_S}{1 - \Gamma_S}$ (3)

Combining Eqs. (1), (2), (3) we can write

$$V_1^+ = \frac{V_S}{2} \frac{1 - \Gamma_S}{1 - \Gamma_S \Gamma_{in}} \quad (11.4)$$

If peak values are assumed for all voltages, the average power delivered to the amplifier is

$$P_{in} = |a_1|^2 - |b_1|^2 = \frac{|V_1^+|^2}{2Z_0}\left(1 - |\Gamma_{in}|^2\right)$$

$$= \frac{V_S^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \Gamma_{in}|^2}\left(1 - |\Gamma_{in}|^2\right) \quad (11.5)$$

Power delivered to the load $= |b_2|^2 - |a_2|^2 = \dfrac{|V_2^-|^2}{2Z_0}\left(1 - |\Gamma_L|^2\right)$ (4)

We have previously derived (on p. 3 of the handout)

$$\frac{b_2}{a_1} = \frac{V_2^-}{V_1^+} = \frac{S_{21}}{1 - S_{22}\Gamma_L} \quad (5)$$

Thus power delivered to the load

$$P_L = \frac{|V_1^+|^2}{2Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2} = \frac{V_S^2}{8Z_0} \frac{|1 - \Gamma_S|^2 |S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_S \Gamma_{in}|^2} \quad (11.7)$$

The power gain can now be written in terms of S-parameters of the amplifier

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} \quad (11.8)$$

Where Γ_{in} is given by Eq. 11.3a.

Note that the power gain obtained in Eq.(11.8) is quite complex, involving as it does, Γ_L, also for Γ_{in} and the S-parameters also in Γ_{in} from Eq. 11.3

The maximum power available from the source (deliverable to the input) is for the condition $\Gamma_{in} = \Gamma_s^*$. Under this input conjugate matching condition, the maximum input power P_{avs} can be written from Eq.(11.5)

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_s^*} = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{(1-|\Gamma_s|^2)} \quad (11.9)$$

Similarly the output power available from the amplifier is that delivered to a matched load for which $\Gamma_L = \Gamma_{out}^*$

From Eq. 11.7

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2}{8Z_0}\frac{|S_{21}|^2(1-|\Gamma_{out}|^2)|1-\Gamma_s|^2}{|1-S_{22}\Gamma_{out}^*|^2\,|1-\Gamma_s\Gamma_{in}|^2} \quad (11.10)$$

However, in Eq. 11.10, Γ_{in} must be evaluated for the output load $\Gamma_L = \Gamma_{out}^*$.

For this conjugate matched output condition

$$|1-\Gamma_s\Gamma_{in}|^2\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2} \quad (6)$$

Substituting (6) into Eq.(11.10) gives maximum available output power

$$P_{avn} = \frac{|V_s|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)} \quad (11.11)$$

From Eqs. 11.11 and 11.9, the available power gain can be written as

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)} \quad (11.12)$$

From Eqs.(11.7) and (11.9), the transducer power gain is

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S \Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2} \qquad (11.13)$$

Special case I

For a special case where both input and output are matched for zero reflection i.e $\Gamma_L = \Gamma_S = 0$, Eq.(11.13) reduces to

$$G_T \Big|_{\Gamma_L=\Gamma_S=0} = |S_{21}|^2 \qquad (11.14)$$

Special case II. Unilateral transducer power gain G_{TU} can be obtained if $S_{12} = 0$

For the case $S_{12} \cong 0$

From eq. 11.3a, $\Gamma_{in} = S_{11}$

From Eq. 11.13

$$G_{TU} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2 |1-S_{22}\Gamma_L|^2} \qquad (11.15)$$

For the example 11.1 on p. 539 text

$$G_{TU} = \frac{(2.05)^2 (1-(0.429)^2)(1-(0.25)^2)}{|1+0.429 \times 0.45 \angle 153|^2 |1+0.25 \times 0.4 \angle -150|^2}$$

$$= \boxed{4.654}$$

pp. 537-539 text — Calculation of Power Gain Expressions Using alternative procedure

a_1, b_2

From Mason's formula

$$\frac{b_2}{a_1} = \frac{S_{21}}{1 - S_{22}\Gamma_L} \quad (1)$$

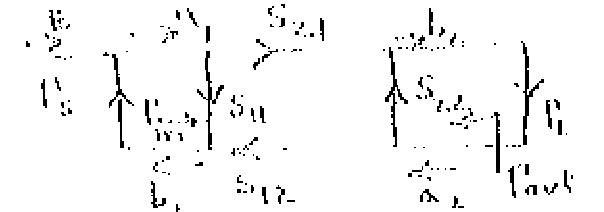


Fig. 1 Flow graph of the amplifier of Fig. 11.1 with source and load mismatch

1. Power Gain G_T = Power delivered to the load / Power input to the amplifier

from eq. (1)

$$= \frac{b_2 b_2^*(1-|\Gamma_L|^2)}{a_1 a_1^* - b_1 b_1^*} = \frac{b_2 b_2^*(1-|\Gamma_L|^2)}{a_1 a_1^*(1-|\Gamma_{in}|^2)} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{in}|^2)}$$

Note that $b_1 = \Gamma_{in} a_1$ (2) (11.8)

2. Maximum power available from the source is for matching condition $\Gamma_{in} = \Gamma_S^*$

from (5)

$$P_{avs} = a_1 a_1^* - b_1 b_1^* = a_1 a_1^*(1-|\Gamma_{in}|^2) = \frac{E E^*(1-|\Gamma_{in}|^2)}{|1-\Gamma_S\Gamma_{in}|^2} \quad (3)$$

$$a_1 = E + \Gamma_S b_1 = E + \Gamma_S \Gamma_{in} a_1 \quad (4)$$

from (2)

Thus $$a_1 = \frac{E}{1-\Gamma_S\Gamma_{in}} \quad (5)$$

Max. Power delivered to the load (for conjugate matched condition $\Gamma_L = \Gamma_{out}^*$)

$$P_{avn} = b_2 b_2^*(1-|\Gamma_L|^2) = a_1 a_1^* \frac{|S_{21}|^2}{|1-S_{22}\Gamma_L|^2} = \frac{E E^* |S_{21}|^2 (1-|\Gamma_{out}|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}$$

from (5) (11.10)

The rest of the steps are the same as on p. 539 of the text

Available power gain $$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)} \quad (11.12)$$

Transducer power gain $$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} \quad (11.13)$$

For the condition when both input and output are well matched

$\Gamma_L = \Gamma_S = 0$

Transducer power gain $G_T = |S_{21}|^2$ (11.14)

Expressions for power gain of an amplifier (see pp. 536–540 of the Text)

We would calculate gain of the amplifier under four different scenarios.

1. Power Gain G

$$G = \frac{\text{Power delivered to the load}}{\text{Power input}} = \frac{b_2 b_2^*(1-|\Gamma_L|^2)}{|a_1|^2-|b_1|^2 \Leftarrow a_1 a_1^* - b_1 b_1^*} \quad (11.8)$$

2. Available Power Gain G_A

$$G_A = \frac{\text{max. power delivered to the load}\Big|_{\Gamma_L = \Gamma_{out}^*} = P_{avn}}{P_{avs}}$$

$P_{avs} \rightarrow$ max power available from the source / input to the amplifier $\Big|_{\Gamma_{in} = \Gamma_S^*}$

i.e. $\frac{\text{max. power output under output conjugate matched condition}}{\text{max power input under conjugate matched condition at input}}$

$P_{avs} \rightarrow$

This is also called available power gain G_A and is given by Eq. (11.12) of the Text

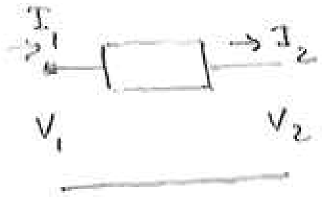
3. Transducer power gain $G_T = \frac{P_L \text{ delivered to the load with } \Gamma_L}{P_{avs} \text{ (When input is conjugate matched)}}$

for the condition that input and output are "well matched".

4. Unilateral power gain $\rightarrow G_{TU} = G_T\Big|_{S_{12}=0} = |S_{21}|^2$ (11.14)

ABCD parameters

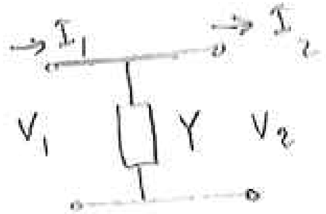
p. 183



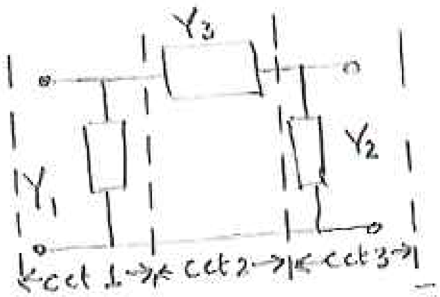
$$V_1 = A V_2 + B I_2$$
$$I_1 = C V_2 + D I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \qquad D = 1$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{Y_3} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}\begin{bmatrix} 1 + \frac{Y_2}{Y_3} & \frac{1}{Y_3} \\ Y_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{Y_2}{Y_3} & \frac{1}{Y_3} \\ Y_1\left(1 + \frac{Y_2}{Y_3}\right) + Y_2 & \frac{Y_1}{Y_3} + 1 \end{bmatrix}$$

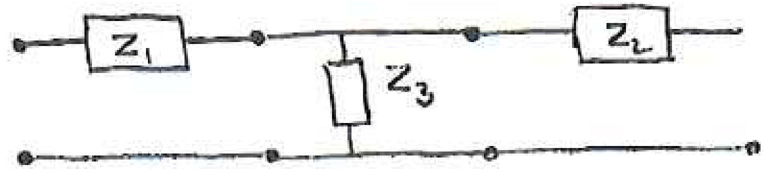
Note that $AD - BC = \left(1 + \frac{Y_2}{Y_3} + \frac{Y_1}{Y_3} + \frac{Y_1 Y_2}{Y_3^2}\right) - \left(\frac{Y_1}{Y_3}\left(1 + \frac{Y_2}{Y_3}\right) + \frac{Y_2}{Y_3}\right)$

$= 1$

for all reciprocal circuits with bilateral performance

$$Z_{12} = Z_{21}; \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}; \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

ABCD Parameters of the Circuit in item 6 of Table 4.1



This circuit may be considered to be composed of 3 Circuits in cascade, as sketched above.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_3} & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ \frac{1}{Z_3} & \frac{Z_2}{Z_3}+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{Z_1}{Z_3} & Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \\ \frac{1}{Z_3} & \frac{Z_2}{Z_3}+1 \end{bmatrix}$$

You may also be able to derive the ABCD parameters using the defining relationships given in (4.63)

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \text{etc.}$$

For homework problem, this basic approach should be followed.