Sample Problem on Rectangular Waveguide (Similar to Prob. 15 of HW; also see Ex. 3.1 of the Text)

a. Select a rectangular waveguide capable of propagating signals at 10 GHz.

From the Table of commercially available rectangular waveguides on p. 127 of my notes, any of the following three waveguides may be used:

	$f_c\big\vert_{TE_{10}} = \frac{c}{2a}$	Recommended frequency range for TE_{10} mode
WR 112	5.259 GHz	7.05 — 10.0 GHz
WR 90	6.557	8.20 — 12.4 GHz
WR 75	7.868	10.00 — 15.00 GHz

Of the three waveguides WR 90 is the best compromise in that it has lower attenuation than WR75 (which is the smallest waveguide for this frequency) and is not as bulky as WR112.

b. List in ascending order the frequencies of the five lowest order modes.

From Eq. 3.84 of the Text

$$f_{c_{mn}} = \frac{c_\epsilon}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

For an air-filled WG $\quad c_\epsilon = c = 3 \times 10^8$ m/sec

For WR90: $\quad a = 0.9'' = 2.286 \times 10^{-2}$ m ; $\quad b = 0.4'' = 1.016 \times 10^{-2}$ m

In order of ascending cutoff frequencies, the five lowest cutoff frequency modes are:

		f_c
TE_{10}	$m=1$, $n=0$	$\frac{c}{2a} = 6.557$ GHz
TE_{20}	$m=2$, $n=0$	$\frac{c}{a} = 13.114$ GHz
TE_{01}	$m=0$, $n=1$	$\frac{c}{b} = 14.764$ GHz
TE_{11}, TM_{11}	$m=1$, $n=1$	$\frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 16.156$

c. For the lowest order (single mode) propagation of TE_{10} mode

$$1.25\,\frac{c}{2a} \le f \le 0.95\, f_c\big\vert_{TE_{20}} \Rightarrow 8.2 \le f \le 12.4 \text{ GHz}$$

c. $$\lambda_g\Big|_{10\,GHz} = \frac{2\pi}{\beta} = \frac{\lambda_0/\sqrt{\epsilon_r}}{\sqrt{1-\left(\frac{f_c^2}{f^2}\right)}} = \frac{3}{\sqrt{1-\left(\frac{6.557}{10}\right)^2}} = 3.975 \text{ cm}$$

$$\lambda_g\Big|_{7.5\,GHz} = \frac{4}{\sqrt{1-\left(\frac{6.557}{7.5}\right)^2}} = 8.24 \text{ cm}$$

d. From Eq. 3-96 on p. 111 of the Text (see also Eqs. on p. 26 of my notes)

$$\alpha_c\Big|_{dB/m} = \frac{R_s}{b\,\eta_\epsilon}\;\frac{f_c}{f}\;\frac{\left[\frac{f^2}{f_c^2}+\frac{2b}{a}\right]}{\sqrt{\frac{f^2}{f_c^2}-1}} \times 8.686$$

to convert nepers/m into dB/m

Assuming Aluminum to be the material of the waveguide

$$R_s = 1.988\sqrt{\frac{f_{MHz}}{\sigma}}$$ from p. 27 of the Text

$$= 1.018\times 10^{-2}\ \Omega$$

$$\alpha_c\Big|_{10\,GHz} = 0.1337 \text{ dB/m}$$

e. power lost in a waveguide for a waveguide length of 3 m

Attenuation $= 0.1337\times 3 \simeq 0.4$ dB

Power output $= P_{in}\, 10^{-0.4/10} = 0.912\, P_{in}$

Thus 8.8 % of the input power is lost over a WG length of 3 m.

f. From Eq. 3.92 of the Text (see also Eq. on p. 26 of my notes)

$$P_{10} = \frac{ab}{4Z_{TE}}(E_{max})^2$$

For an input power of 100 KW at 10 GHz (air-filled WG)

$$Z_{TE} = \frac{\eta_\epsilon}{\sqrt{1-(f_c^2/f^2)}} = 499.5\ \Omega \quad (3.92)$$

$E_{max} = 9.275\times 10^3$ V/cm (this is less than the dry air breakdown field strength of 29000 V/cm)

g. $$P_{BD} = 3.6\; a\big|_{inch}\; b\big|_{inch} \sqrt{1-(f_c^2/f^2)} = 0.978 \text{ MW}$$

Ex. Find the load impedance if VSWR and location of voltage maximum (or minimum) is known.

say $\quad VSWR = 2.0 \quad |\Gamma| \rightarrow |S_{11}| = \dfrac{VSWR-1}{VSWR+1}$

location of voltage maximum (impedance ~~minimum~~ maximum) $= 0.3\,\lambda_g$ from the load.
1.1925 cm @ 10 GHz

From Smith chart $\quad z_L = 0.56 - j\,0.24$

$\rho_L = 0.32\,\angle{-144^\circ} \quad \leftarrow \quad \dfrac{z_L - 1}{z_L + 1}$

an open ended WG acting as a radiator presents an effective load to the WG.

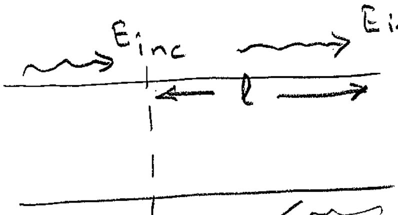
$$Z_L = z_L Z_{TE} = 499.5\,(0.56 - j\,0.24)$$
$$= 279.7 - j\,119.9\ \Omega$$

Ex. calculate the magnitude of $\vec{E}$ at the load or the open end of the WG

say $l = 0.3\,\lambda_g$

for $P_{in} = 100$ kW

$E_{inc} = 9.275 \times 10^5$ V/m peak

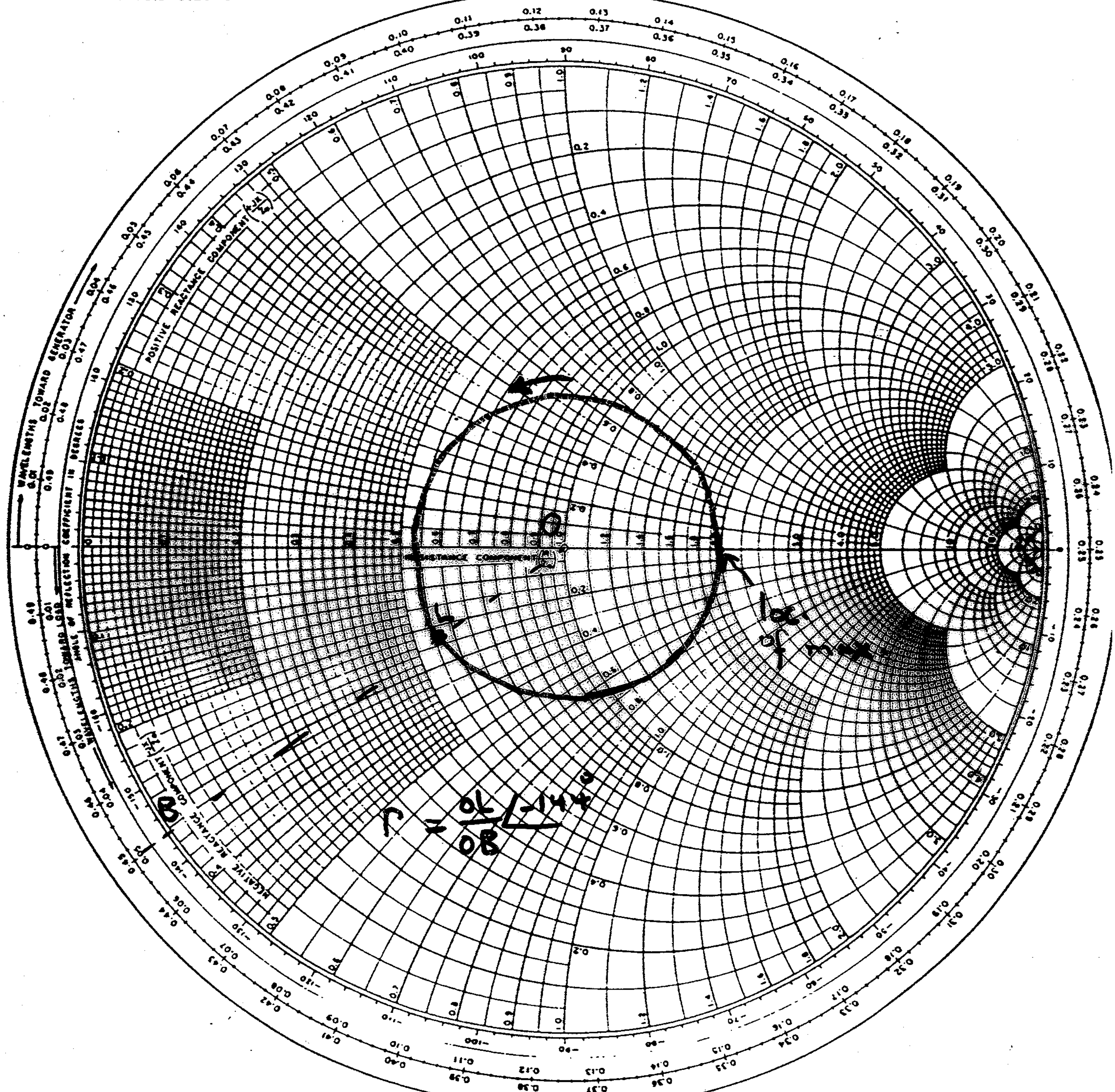


$E_r = (\rho\,e^{-j\phi})(E_{inc}\,e^{-j\beta l})$

$$\vec{E}\Big|_{\text{open end}} = E_{inc}\,e^{-j\beta l}\left[1 + |\Gamma|\,e^{-j\phi}\right]$$

$$\left|\,\vec{E}\Big|_{\text{open end}}\right| = E_{inc}\,\left|1 + 0.32\,e^{-j144^\circ}\right|$$
$$= 0.7646\,E_{inc} = 7.09 \times 10^5 \text{ V/m peak}$$

"SMITH CHART"


POSITIVE REACTANCE COMPONENT
NEGATIVE REACTANCE COMPONENT
WAVELENGTHS TOWARD GENERATOR
B
Γ = OL/OB ∠-144°