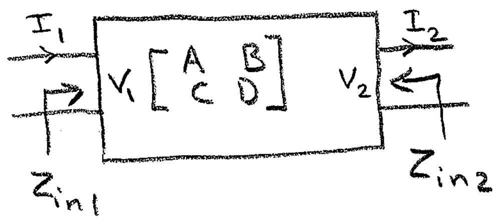
Notes for
Chapter 8 – 2008

Chapter 8 — Microwave Filters

Section 8.2 Text (p.378)



Z_{i1} = input impedance at port ① when port ② is terminated with Z_{i2}

Z_{i2} = input impedance at port 2 when port 1 is terminated with Z_{i1}

From Eq. 4.63 p.183 of the Text, the port voltages and currents are related as follows:

$$V_1 = A V_2 + B I_2 \qquad (8.22a)$$
$$I_1 = C V_2 + D I_2 \qquad (8.22b)$$

$$Z_{in1} = \frac{V_1}{I_1} = \frac{A V_2 + B I_2}{C V_2 + D I_2} = \frac{A Z_{i2} + B}{C Z_{i2} + D} \qquad (8.23)$$

Note that $V_2 = I_2 Z_{i2}$

From Eq. 4.67 p.186 of the Text, $AD - BC = 1$ for passive networks

From Eqs. 8.22 a, b, we can derive the following:

$$V_2 = D V_1 - B I_1 \qquad (8.24a)$$
$$I_2 = -C V_1 + A I_1 \qquad (8.24b)$$

$$Z_{in2} = -\frac{V_2}{I_2} = -\frac{D V_1 - B I_1}{-C V_1 + A I_1} = \frac{D Z_{i1} + B}{C Z_{i1} + A} \qquad (8.25)$$

We desire that $Z_{in1} = Z_{i1}$ and $Z_{in2} = Z_{i2}$

From Eqs. (8.23) and (8.25) we can therefore write

$$Z_{i1}(C Z_{i2} + D) = A Z_{i2} + B \qquad (8.26a)$$
$$Z_{i1} D - B = Z_{i2}(A - C Z_{i1}) \qquad (8.26b)$$

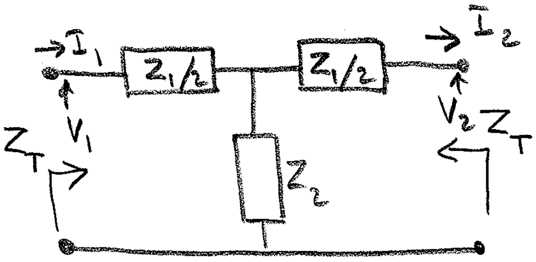
Solving for Z_{i1}, Z_{i2} gives

$$Z_{i1} = \sqrt{\frac{AB}{CD}} \quad (8.27a) \; ; \quad Z_{i2} = \frac{D}{A} Z_{i1} = \sqrt{\frac{BD}{AC}} \quad (8.27b)$$

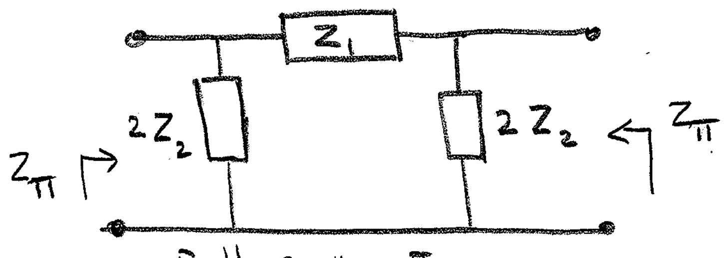
$\boxed{\text{If the network is symmetric, then } A = D \text{ and } Z_{i1} = Z_{i2}}$

See Table 4-1 $A = D$ for Symmetric Networks

p. 381 Table 8.1 Image Parameters for T and π Networks



Full section T



Full section π

From the expressions for elements 5 & 6 in Table 4.1 (p. 185 Text)

$$A = 1 + \frac{Z_1}{2Z_2} \qquad\qquad A = 1 + \frac{Z_1}{2Z_2}$$

$$B = Z_1 + \frac{Z_1^2}{4Z_2} \qquad\qquad B = Z_1$$

$$C = \frac{1}{Z_2} \qquad\qquad C = \frac{1}{Z_2} + \frac{Z_1}{4Z_2^2}$$

$$D = 1 + \frac{Z_1}{2Z_2} \qquad\qquad D = 1 + \frac{Z_1}{2Z_2}$$

$$Z_T = \sqrt{\frac{AB}{CD}} \quad 8.27(a)$$

$$= \sqrt{Z_1 Z_2}\sqrt{1 + \frac{Z_1}{4Z_2}}$$

$$Z_\pi = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1 Z_2}{4Z_2^2}}}$$

$$\cosh\gamma = \sqrt{AD} = 1 + \frac{Z_1}{2Z_2} \quad (8.31)$$

$$= \frac{e^{\gamma} + e^{-\gamma}}{2}$$

$$\boxed{\gamma = \alpha + j\beta}$$

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2} \quad (8.31)$$

$$\boxed{\frac{V_2 I_2}{V_1 I_1} = e^{-2\gamma}}$$

For pass band $\quad \gamma = j\beta \quad ; \quad \alpha = 0$

$$\cos\beta = 1 + \frac{Z_1}{2Z_2} = \cos^2(\beta/2) - \sin^2(\beta/2) = 1 - 2\sin^2(\beta/2)$$

$$= 1 + \frac{Z_1}{2Z_2}$$

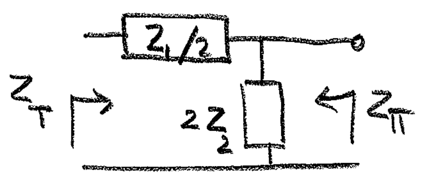
For β to be real $\quad -1 \leq \cos\beta \leq 1$

$$-2 \leq \frac{Z_1}{2Z_2} \leq 0 \quad \text{or} \quad -1 < \frac{Z_1}{4Z_2} \leq 0$$

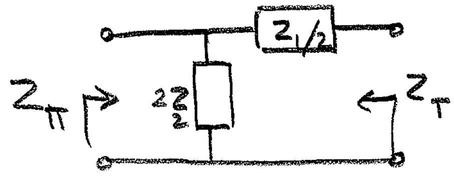
For pass band

$$\beta = \cos^{-1}\left(1 + \frac{Z_1}{2Z_2}\right) = \pm 2\sin^{-1}\left(\sqrt{-\frac{Z_1}{4Z_2}}\right)$$

Bisected half-section Filters



$$\boxed{Z_T Z_\pi = Z_1 Z_2}$$



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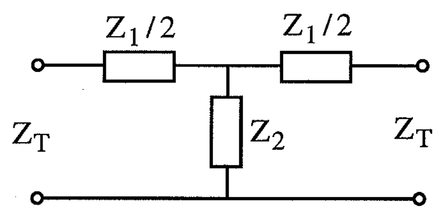
Some General Relationships for Filter Design by Image Parameter Method

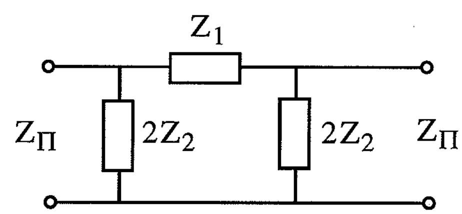
See Table 8.1 p. 381

T Network



$$Z_T = \sqrt{Z_1 Z_2}\,\sqrt{1+\frac{Z_1}{4Z_2}}$$

Π Network



$$Z_\Pi = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1+\frac{Z_1}{4A_2}}} = \frac{Z_1 Z_2}{Z_T}$$

For pass band: Both Z_T and Z_Π **are purely resistive**.

$$-1 \le \frac{Z_1}{4Z_2} \le 0$$

$$\beta = \pm\, 2 \sin^{-1}\left(-\frac{Z_1}{4Z_2}\right)^{1/2} \text{ radians}$$

For stop band: Both Z_T and Z_Π **are purely reactive.**

$$\frac{Z_1}{4Z_2} > 0; \qquad \alpha = 2 \sinh^{-1}\left(\frac{Z_1}{4Z_2}\right)^{1/2}$$

$$\frac{Z_1}{4Z_2} < -1; \qquad \alpha = \cosh^{-1}\left|1+\frac{Z_1}{2Z_2}\right|$$

$$= 2 \cosh^{-1}\left(\frac{-Z_1}{4Z_2}\right)^{1/2}$$

Constant-k Filters

p. 380, also Table 8.2, p. 387

Nominal terminating resistance $R = (Z_1 Z_2)^{1/2} \Rightarrow k$

Low pass

$Z_1 = j\omega L \qquad R_o = \sqrt{\frac{L}{C}}$

$Z_2 = \frac{1}{j\omega C} \qquad L = \frac{2R_o}{\omega_c}, \quad C = \frac{2}{\omega_c R_o}$

$\omega_c = \frac{2}{\sqrt{LC}} \qquad \beta = 2\sin^{-1}\left(\frac{\omega}{\omega_c}\right)$

High pass

$Z_1 = \frac{1}{j\omega C} \qquad R_o = \sqrt{\frac{L}{C}}$

$Z_2 = j\omega L \qquad L = \frac{R}{2\omega_c}, \quad C = \frac{1}{2\omega_c R_o}$

$\omega_c = \frac{1}{2\sqrt{LC}} \qquad \beta = 2\sin^{-1}\left(\frac{\omega_c}{\omega}\right)$

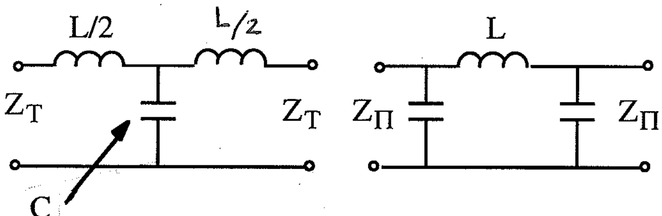


Fig. 8.9

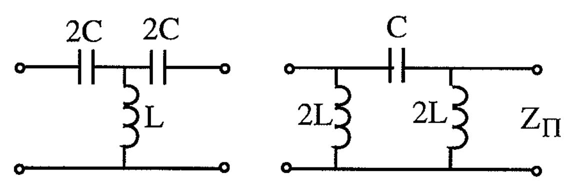


Fig. 8.11

$$Z_T = R_o\left(1 - \frac{\omega^2}{\omega_c^2}\right)^{1/2}; \quad Z_\Pi = \frac{R_o}{\left(1 - \frac{\omega^2}{\omega_c^2}\right)^{1/2}}$$

$$Z_T = R_o\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{1/2}; \quad Z_\Pi = \frac{R_o}{\left(1 - \frac{\omega_c^2}{\omega^2}\right)^{1/2}}$$

Disadvantages of Constant-k Filters

1. Actual termination resistances Z_T, Z_Π vary greatly over the pass band.
2. The attenuation constant α varies undesirably slowly over the stop band.

m-Derived Filters

p. 383

The m-derived filters are a considerable improvement for both of the enumerated disadvantages of the constant-k filters.

T Network (series m-derived)

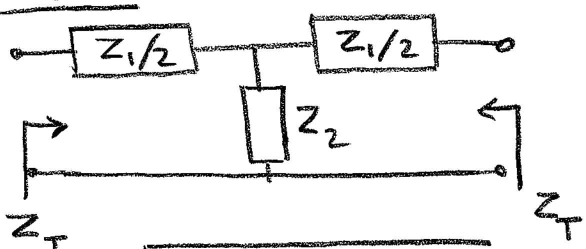
$Z_1' = mZ_1 \qquad (8.39)$

Π Network (shunt m-derived)

$Z_2' = \frac{Z_2}{m}$

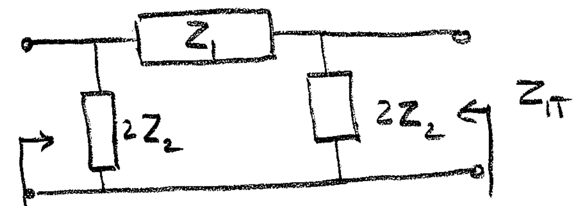
Constant-k Filters

T – Section



$$Z_T = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

Π – Section

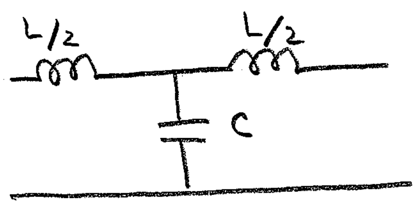


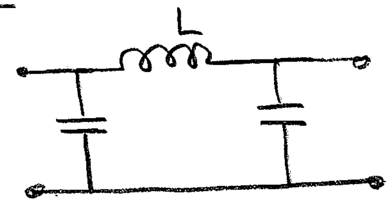
$$Z_\pi = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} = \frac{Z_1 Z_2}{Z_T}$$

For pass band $\quad 0 \le -\dfrac{Z_1}{4Z_2} \le 1 \;;\; \beta = \pm 2 \sin^{-1}\sqrt{-\dfrac{Z_1}{4Z_2}}$

For stop band $\quad \left|\dfrac{Z_1}{4Z_2}\right| > 1 \;;\; \alpha = 2 \sinh^{-1}\left(-\dfrac{Z_1}{4Z_2}\right)^{1/2}$

Case I : Low PASS FILTERS

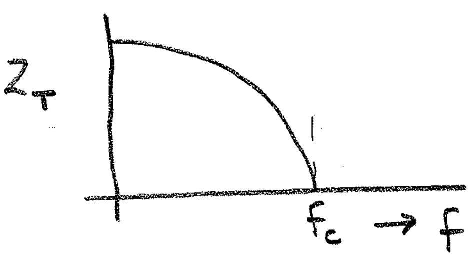


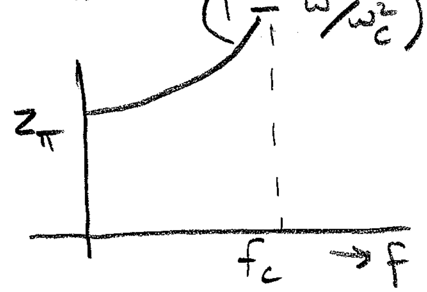


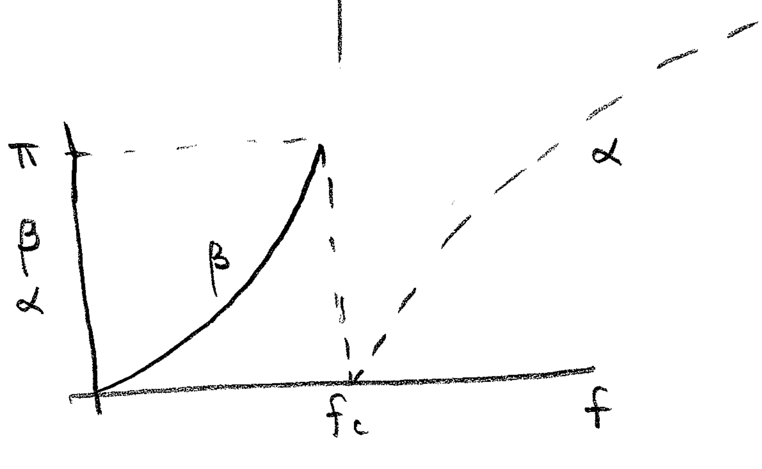
$$L = \frac{2R_0}{\omega_c} \;;\quad C = \frac{2}{\omega_c R_0} \;;\quad \omega_c = \frac{2}{\sqrt{LC}}$$

$$\beta = 2 \sin^{-1}(\omega/\omega_c)$$

$$Z_T = R_0\left(1 - \frac{\omega^2}{\omega_c^2}\right)^{1/2} \qquad Z_\pi = \frac{R_0}{\left(1 - \omega^2/\omega_c^2\right)^{1/2}}$$







$$Z_2' = \frac{Z_2}{m} + \frac{1-m^2}{4m} Z_1 \qquad (8.41)$$

$$\left.\frac{1}{Z_\Pi}\right|_{m-\text{derived}} = \left.\frac{1}{Z_\Pi'}\right|_{k-\text{sec tion}}$$

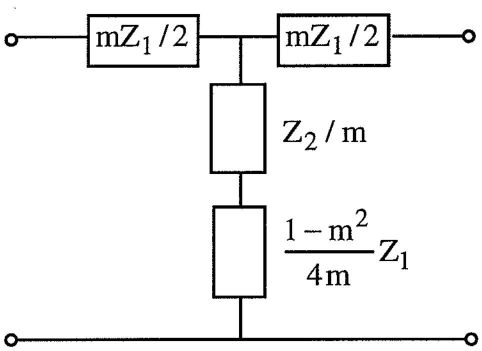
$$\left.\frac{1}{Z_T}\right|_{m-\text{derived}} = \left.\frac{1}{Z_T}\right|_{k-\text{sec tion}}$$

$$\frac{1}{Z_1'} = \frac{1}{mZ_1} + \frac{(1-m^2)/m}{4Z_2}$$

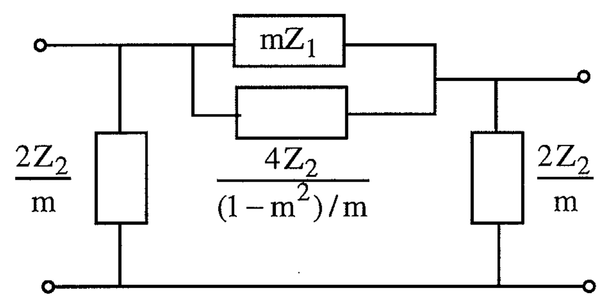
$Z_2' \equiv$ Series combination of $\frac{Z_2}{m}$ and $\frac{1-m^2}{4m} Z_1$

$$\left.Z_\Pi\right|_{m-\text{derived}} = \frac{Z_1' Z_2'}{Z_T} = \frac{Z_1 Z_2 + \frac{1-m^2}{4} Z_1^2}{Z_T}$$

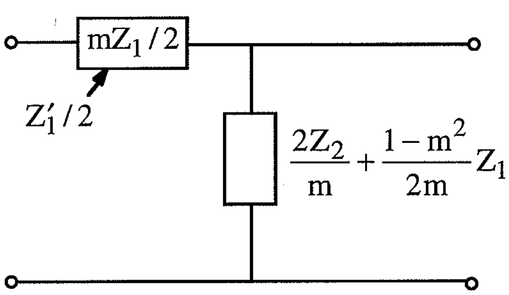
$Z_1' =$ Parallel combination of mZ_1 and $\frac{4Z_2}{(1-m^2)/m}$



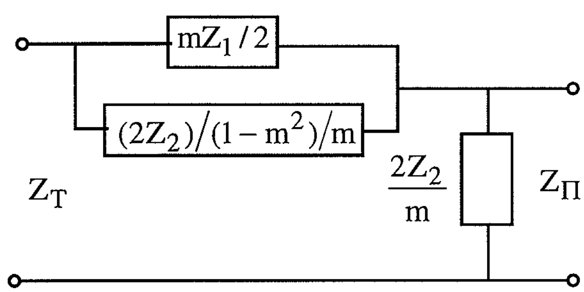
Full section T (Fig. 8.12, p. 383)



Full section Π



Half section T

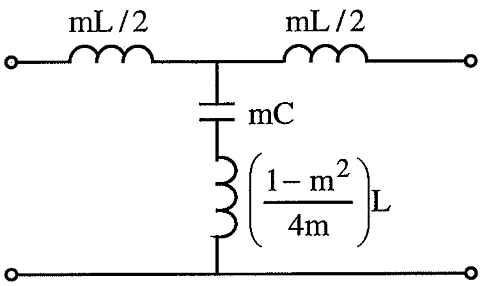


For Low Pass Filters

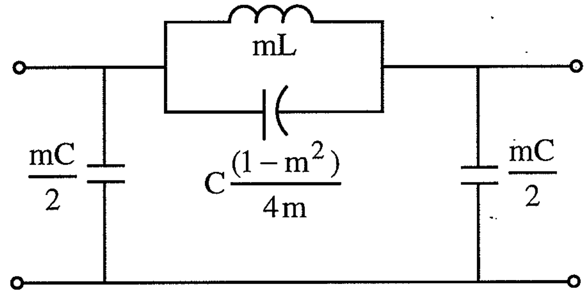
$$Z_1' = j\omega Lm \qquad Z_2' = \frac{1}{j\omega(mC)}$$

$$Z_2' = \frac{1}{j\omega(mC)} + \left(\frac{1-m^2}{4m}\right) j\omega L$$

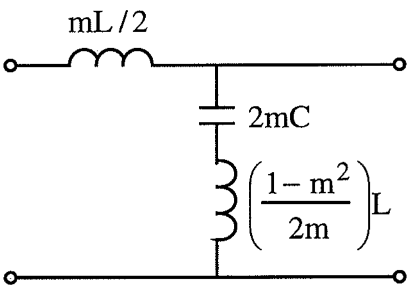
$$\frac{1}{Z_1'} = \frac{1}{j\omega(mL)} + j\omega C \frac{(1-m^2)}{4m}$$



Full section T (Fig. 8.13)

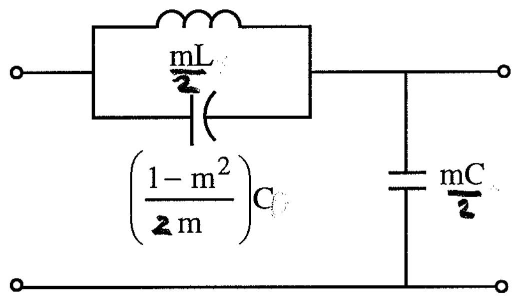


Full section Π



$$\left.\frac{mL}{2}\right|_{text} = mL_k\big|_{handout}$$

$$2C\big|_{text} = C_k\big|_{handout}$$



Half section T (p. 7-1)

Design of Constant-k and m-Derived Low Pass Filters

Example: Select R_o, f_c, f_∞ (for m-derived sections)

$R_o = 50\Omega$; $f_c = 2.0$ GHz; $f_\infty = 2.5$ GHz

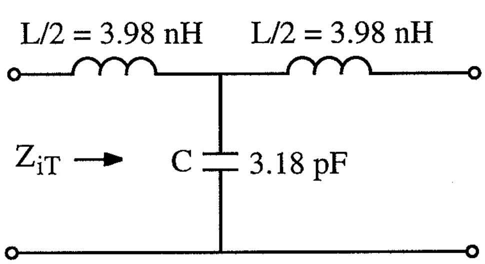
$m = 0.6$

From Eqs. 8.33, 8.34 (p. 380 text)

$$L = \frac{2R_o}{\omega_C} = \frac{100}{2\Pi\times 2\times 10^9} = 7.96 \text{ nH}$$

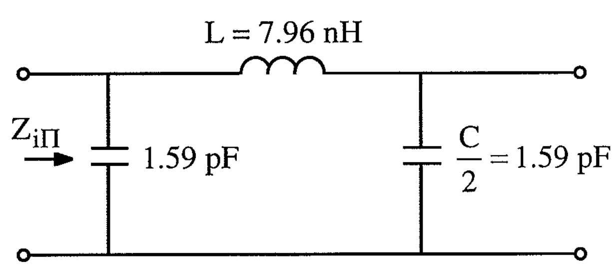
$$C = \frac{2}{\omega_C R_o} = \frac{2}{2\Pi\times 2\times 10^9\times 50} = 3.18 \text{ pF}$$

Constant k-Section Filters



T - section filter

$$Z_{iT} = R_o\sqrt{\left(1-\frac{f^2}{f_c^2}\right)}$$

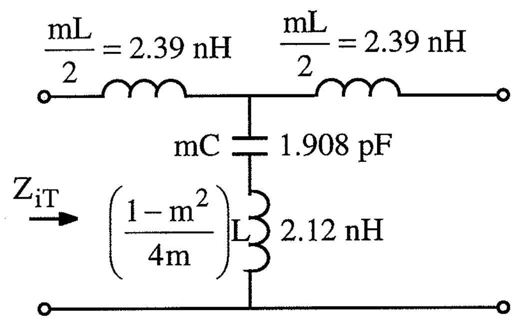


Π - section filter

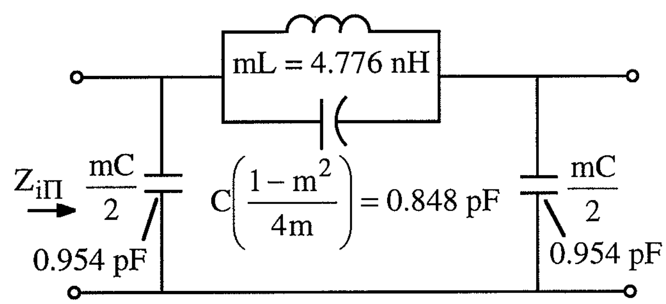
$$Z_{i\Pi} = \frac{R_o}{\sqrt{1-\frac{f^2}{f_c^2}}}$$

m-Derived Filters

From Eq. 8.44, $$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$ $$m = \sqrt{1-\frac{f_c^2}{f_\infty^2}} = \sqrt{1-\left(\frac{2}{2.5}\right)^2} = 0.6$$



Series m-derived filter



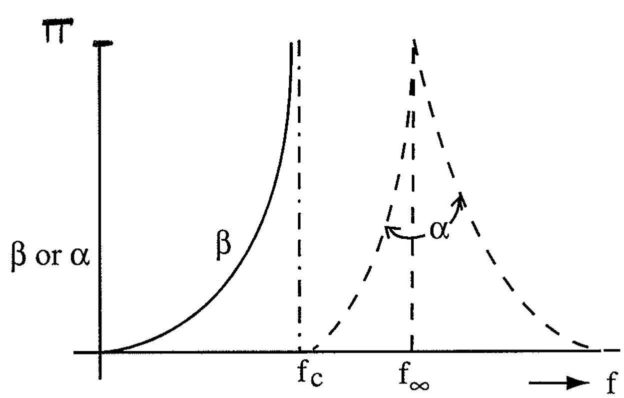
Shunt m-derived low pass filter

$$Z_{iT} = R_o\sqrt{1-\frac{f^2}{f_c^2}}$$

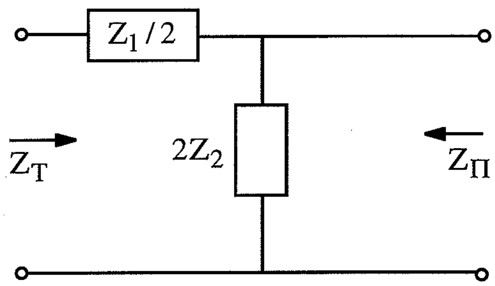
For Stop Band

$$\alpha = 2\cosh^{-1}\left(-\frac{Z_1'}{4Z_2'}\right) \quad \text{for} \quad \frac{Z_1'}{4Z_2'} < -1$$

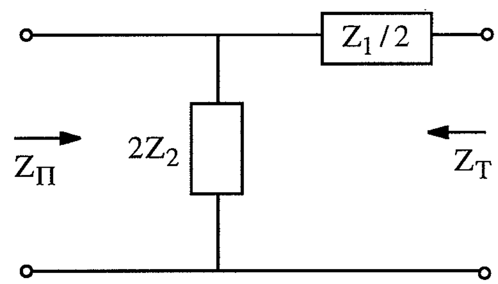
$$= 2\sinh^{-1}\left(\frac{Z_1'}{4Z_2'}\right) \quad \text{for} \quad \frac{Z_1}{4Z_2} > 0$$



Half Section Filters (k-Section)



Half section T

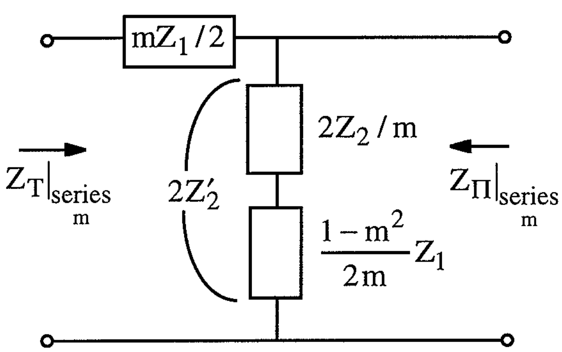


Half section Π

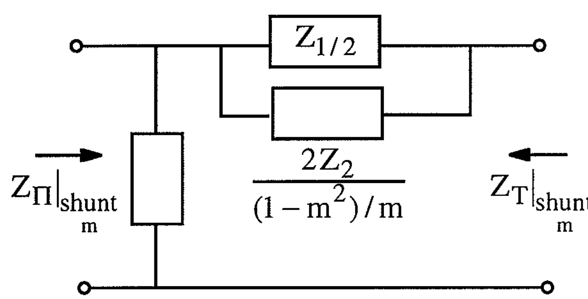
$$Z_T Z_\Pi = Z_1 Z_2$$

m-Derived Half Section Filters

Series m-derived



Shunt m-derived



$$Z_T\Big|_{\substack{\text{series}\\ \text{m-derived}}} = Z_T\Big|_{\text{k-section}} = R_o\sqrt{1-\frac{f_c^2}{f^2}}$$

$$Z_\Pi\Big|_{\substack{\text{shunt}\\ m}} = Z_\Pi\Big|_{\text{k-section}} = \frac{R_o}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$Z_\Pi\Big|_{\substack{\text{series}\\ \text{m-derived}}} = \frac{Z_1' Z_2'}{Z_T} = \frac{Z_1 Z_2 + \left(\frac{1-m^2}{4}\right) Z_1^2}{Z_T}$$

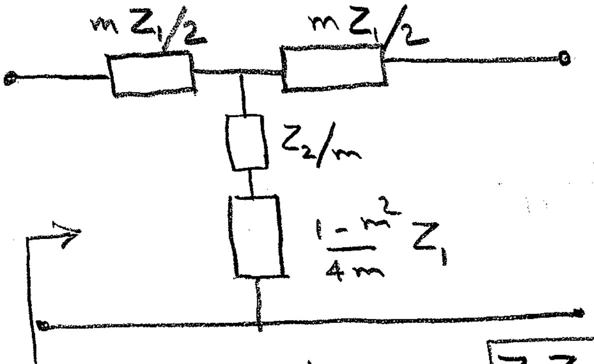
$$Z_T\Big|_{\substack{\text{shunt}\\ m}} = \frac{Z_1' Z_2'}{Z_\Pi\Big|_{\substack{\text{shunt}\\ m}}}$$

m-derived filters

purpose: to get $\alpha \to \infty$ at a frequency f_∞ close to f_c

Series m-derived

$Z_1' = mZ_1$

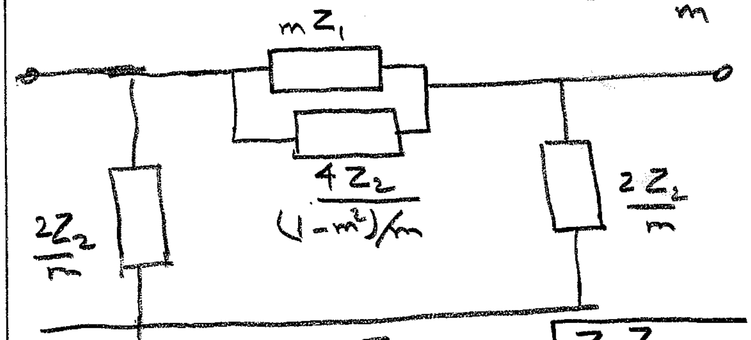


$$Z_T\Big|_{\text{series m-derived}} = Z_T\Big|_{\text{k-section}} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$Z_\pi\Big|_{\text{series m-derived}} = \frac{Z_1' Z_2'}{Z_T}$$

Shunt m-derived

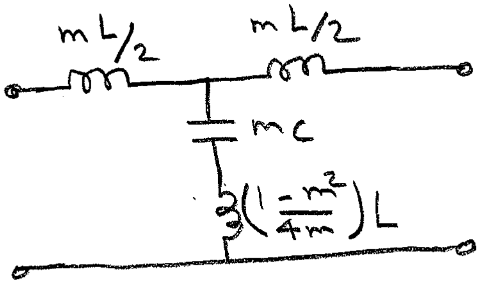
$Z_2' = \frac{Z_2}{m}$

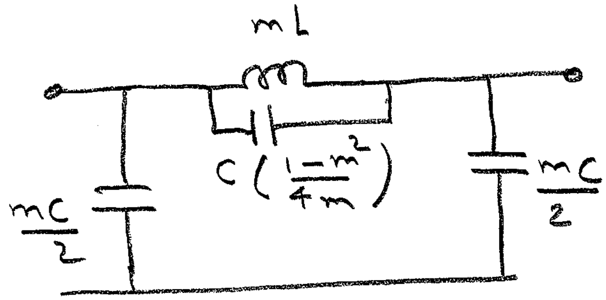


$$Z_\pi\Big|_{\text{shunt m-derived}} = Z_\pi\Big|_{\text{k-section}} = \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}}$$

$$Z_T\Big|_{\text{shunt m-derived}} = \frac{Z_1' Z_2'}{Z_\pi\Big|_{\text{k-section}}}$$

Case I **Low pass-filters**





$$L = \frac{2R_0}{\omega_c}, \quad C = \frac{2}{\omega_c R_0}, \quad \omega_c = \frac{2}{\sqrt{LC}}$$

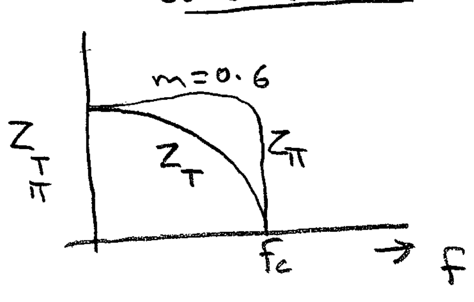
$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

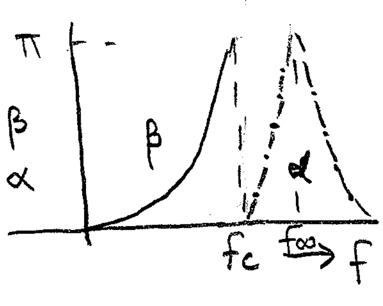
$$Z_T\Big|_{\text{series m-derived}} = Z_T\Big|_{\text{k-section}} = R_0\left(1-\frac{\omega^2}{\omega_c^2}\right)^{1/2}$$

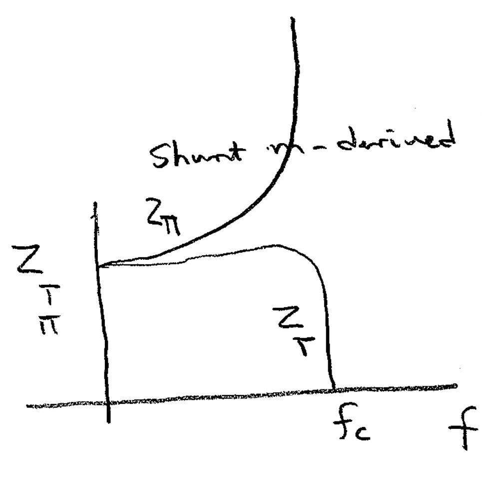
$$Z_\pi\Big|_{\text{series m-derived}} = \frac{Z_1' Z_2'}{Z_T} = \frac{R_0\left(1-\omega^2/\omega_\infty^2\right)}{\sqrt{1-\frac{\omega^2}{\omega_c^2}}}$$

$$Z_\pi\Big|_{\text{shunt m-derived}} = Z_\pi\Big|_{\text{k-section}} = \frac{R_0}{\sqrt{1-\omega^2/\omega_c^2}}$$

Series m-derived







For a high pass filter

$$Z_1 = \frac{1}{j\omega C} \quad ; \quad Z_2 = j\omega L$$

pass band: $-1 \le \frac{Z_1}{4Z_2} \left(= -\frac{\omega_c^2}{\omega^2}\right) \le 0$

@ $\omega = \omega_c$; $\frac{Z_1}{4Z_2} = -1$

for $\omega > \omega_c$; $\frac{Z_1}{4Z_2} = -\frac{\omega_c^2}{\omega^2}$ is between -1 & 0

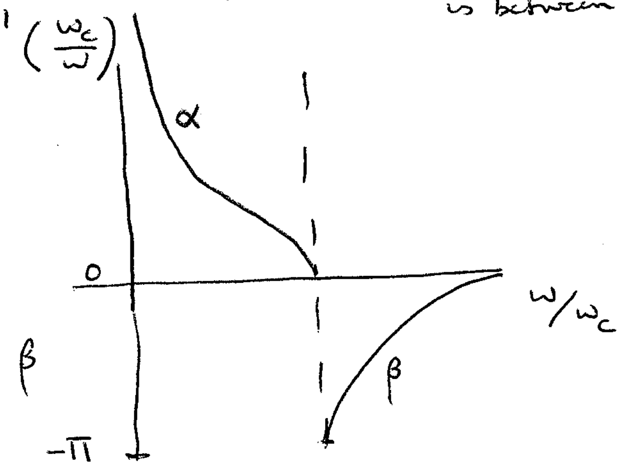
$$\beta = \cos^{-1}\left(1 + \frac{Z_1}{2Z_2}\right) = \cos^{-1}\left(1 - 2\frac{\omega_c^2}{\omega^2}\right)$$

$$= \pm 2 \sin^{-1}\sqrt{\frac{-Z_1}{4Z_2}} = \pm 2 \sin^{-1}\left(\frac{\omega_c}{\omega}\right)$$

Stop band

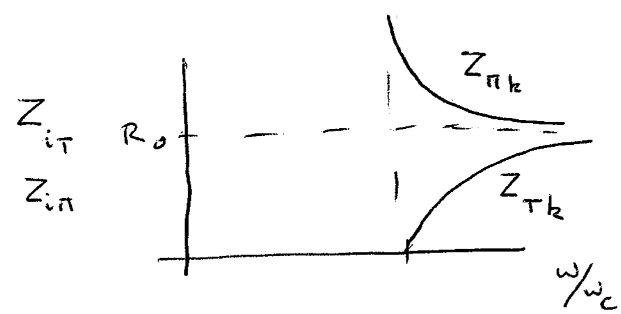
$$\alpha = 2\cosh^{-1}\left(-\frac{Z_1}{4Z_2}\right)^{1/2}$$

$$= 2\cosh^{-1}(\omega_c/\omega)$$

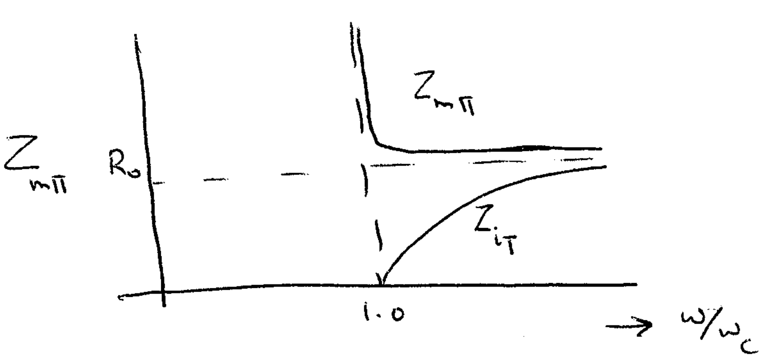


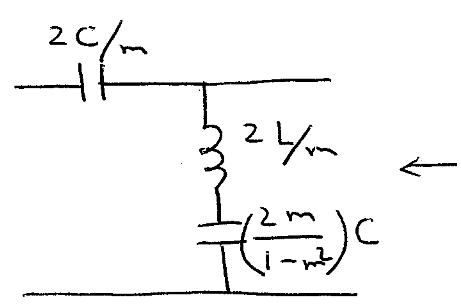
$$Z_{iT} = R_0\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$Z_{i\pi} = R_0 \Big/ \sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$



For series m-derived (see also p. 442 Table 8.2



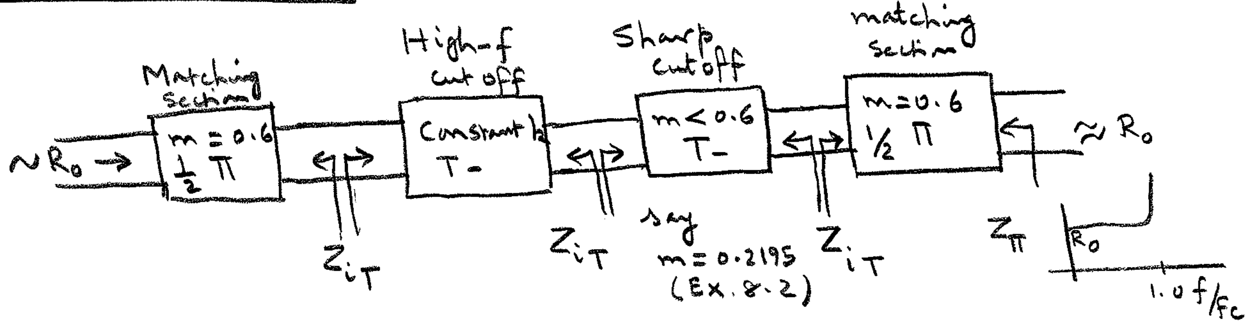


$\frac{1}{2}$ Section π

$$Z_{m\pi} = R_0 \frac{1 - f_\infty^2/f^2}{\sqrt{1 - f_c^2/f^2}}$$

p.387

A Composite Low-Pass Filter



Ex. 8.2 p.388

$f_c = 2.0\ \text{MHz}$

$f_\infty = 2.05\ \text{MHz}$

$f_c' = 3000\ \text{MHz}$

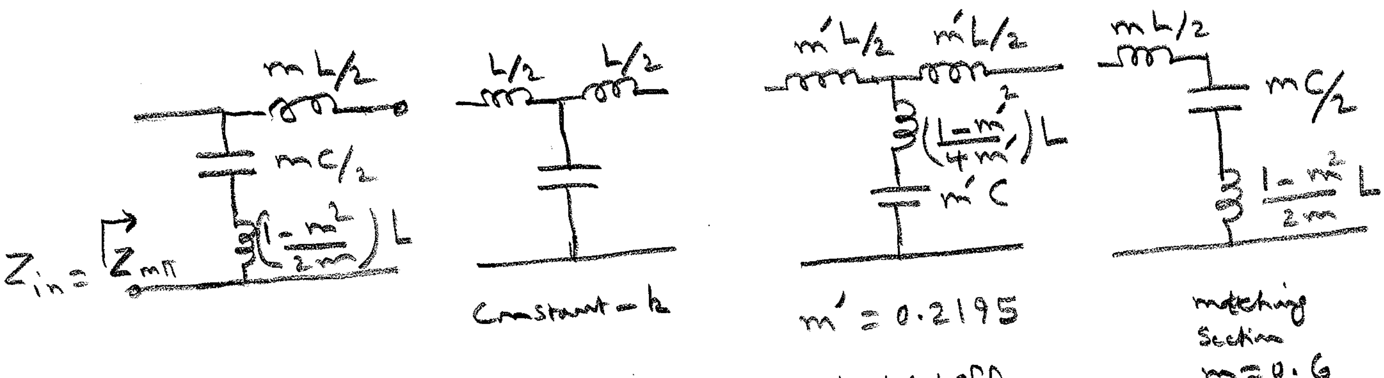
$f_\infty' = 3075\ \text{MHz}$

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

$$m' = 0.2195$$

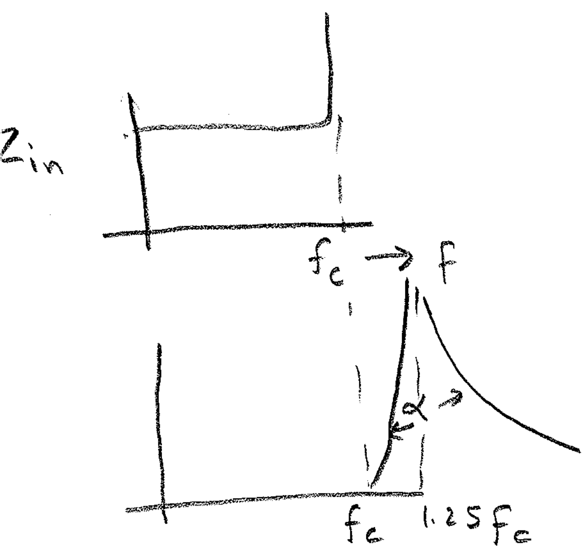
$$Z_{m\pi} = \frac{R_0\left(1-\omega^2/\omega_\infty^2\right)}{\sqrt{1-\omega^2/\omega_c^2}}$$

$$\frac{Z_1' Z_2'}{Z_{mT}}$$



From p. 2 of handout notes

$$L = \frac{2R_0}{\omega_c}\ ;\quad C = \frac{2}{\omega_c R_0}\ ;\quad \omega_c = \frac{2}{\sqrt{LC}}$$



Ex 8.2 p. 388 Text

$f_c = 2\ \text{MHz}$

$f_\infty = 2.05\ \text{MHz}$

$$m = \sqrt{1 - \frac{f_c^2}{f_\infty^2}} = 0.2195$$

$$R_o = \sqrt{\frac{L}{C}} = 75\ \Omega \qquad (8.34)\ \text{p. 433 Text}$$

$$f_c = \frac{1}{2\pi}\frac{2}{\sqrt{LC}} \qquad (8.33)$$

$$L = \frac{2R_o}{\omega_c} = 11.94\ \mu H$$

$$C = \frac{2}{R_o \omega_c} = 2.122\ nF$$

$f_c' = 3000\ \text{MHz}$

$f_\infty' = 3075\ \text{MHz}$

$$m = \sqrt{1 - \frac{f_c'^2}{f_\infty'^2}} = 0.2195$$

$$R_o' = \sqrt{\frac{L'}{C'}} = 50\ \Omega$$

$$f_c' = \frac{1}{2\pi}\frac{2}{\sqrt{L'C'}}$$

$$L' = \frac{2R_o'}{\omega_c'} = \left(\frac{50}{75 \times 1500}\right) \times 11.94 \times 10^3\ nH = 5.31\ nH$$

$$C' = \frac{2}{R_o'\omega_c'} = \left(\frac{75}{50}\right)\frac{1}{1500} \times 2122\ pF = 2.122\ pF$$

Redrawn cct. of Fig. 8.19 p. 388 (divide L's by 2250 and C's by 1000)

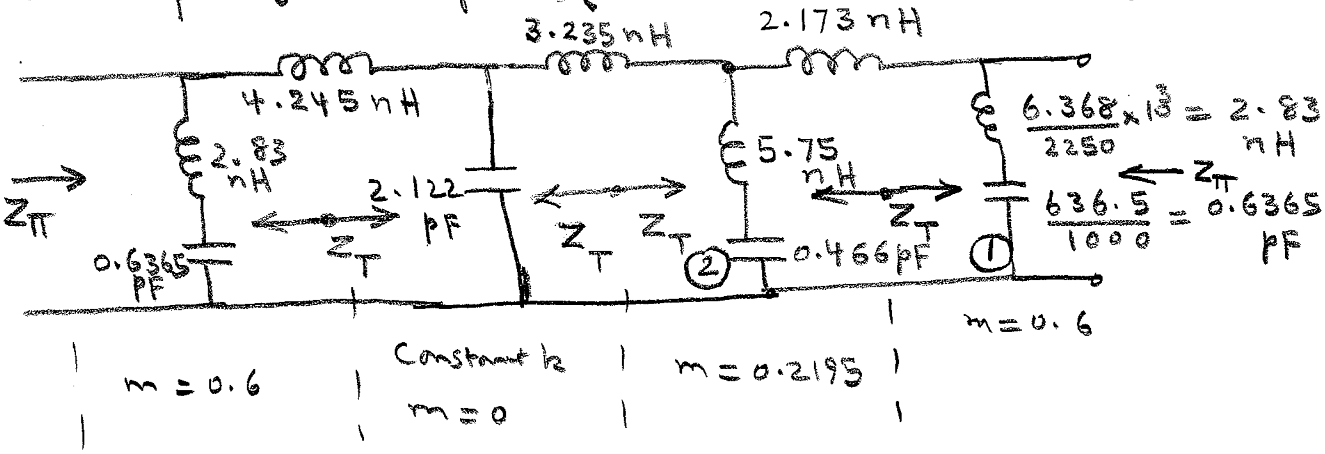


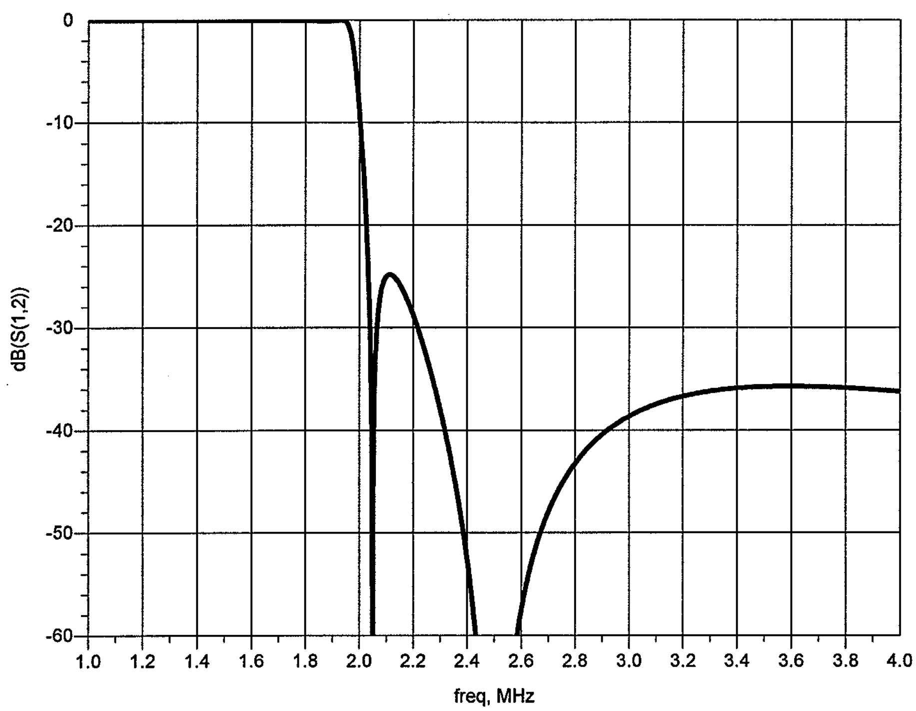
Fig. 8.19 (redrawn) A Composite filter with $f_c' = 3000\ \text{MHz}$; $f_\infty'\big|_{m=0.2195} = 3075\ \text{MHz}$

For the above circuit note the following resonant frequencies for the shunt arms:

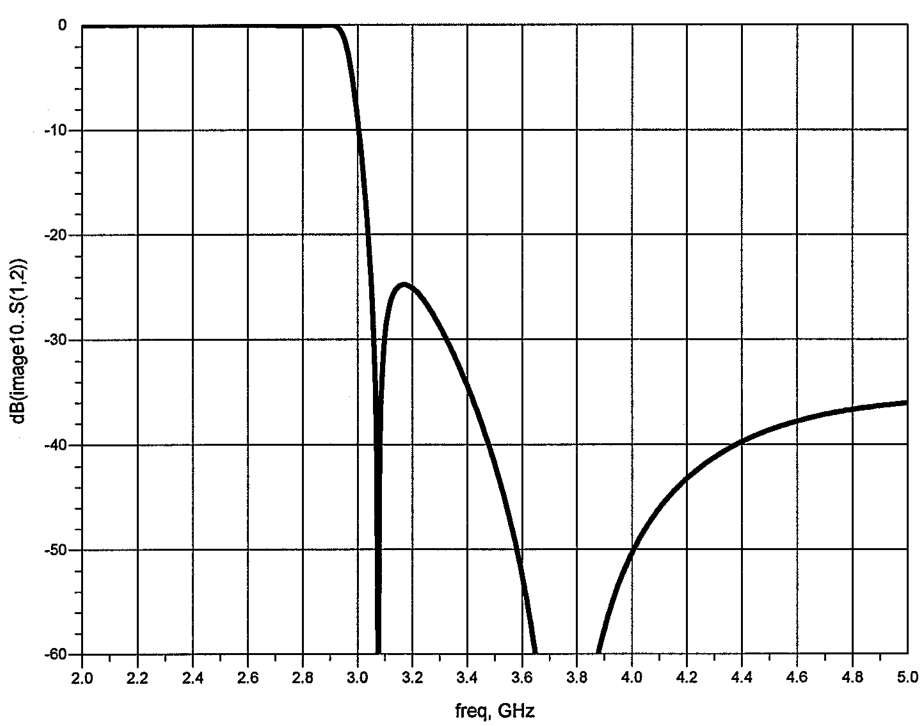
For arm 1 $$f_{r1} = \frac{1}{2\pi \times \sqrt{(2.83 \times 10^{-9})(0.6365 \times 10^{-12})}} = 3.75\ \text{GHz} \quad (m = 0.6)$$

for arm 2 $$f_{r,2} = \frac{1}{2\pi\sqrt{(5.75 \times 10^{-9})(0.466 \times 10^{-12})}} = 3.075\ \text{GHz} \quad (m = 0.2195)$$

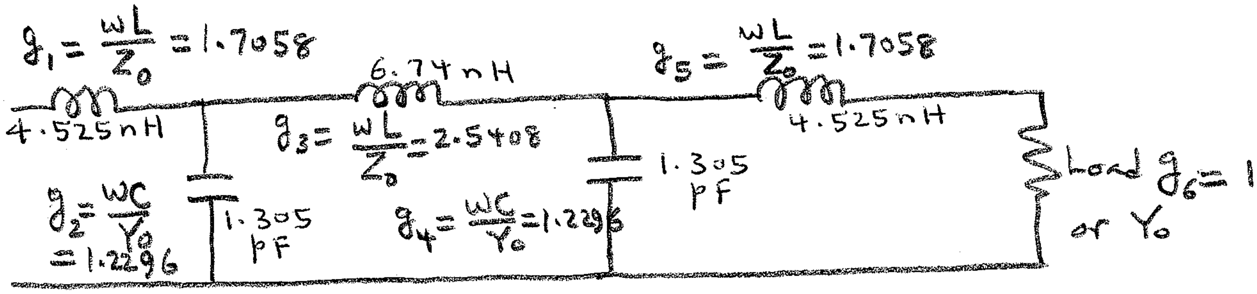
Frequency response for the low-pass filter of Example 8.2
with cut-off frequency of 2 MHz. R0 = 75 ohm.



Frequency response for the scaled microwave filter with cut-off
frequency of 3 GHz, designed on pg. 10 of classnotes. R0 = 50 ohm



Lab. #4 Filter #2 See Section 8.3 Insertion Loss Method
(See also Example 8.3 p. 400 Text)
N = 5 , 0.5 dB, Low Pass Filter (Equal-ripple filter)
For a filter starting and ending with a series inductor (see the suggested configuration for Filter #2 for Lab. 4, Use the circuit of Fig. 8.25(b) p. 393 Text (See parameters g_1 to g_6 in Table 8.4)



See p. 394 $g_K =$ inductance for series inductors $(\omega L / Z_0)$
($k = 1$ to N)
= Capacitance for shunt capacitors $\left(\frac{\omega C}{Y_0}\right)$

For Cutoff frequency $f_c = 3.0$ GHz , $Z_0 = 50\ \Omega$

for $\frac{\omega L}{Z_0} = 1.7058$ $L = 4.525$ nH

$\frac{\omega C}{Y_0} = 1.2296$ $C = 1.305$ pF

$\frac{\omega L}{Z_0} = 2.5408$ $L = 6.740$ nH

As discussed on the following page 12, the inductances can be obtained by using high Z_0 microstrip lines (for $Z_0 = 120\ \Omega$; $L_\ell = 0.521$ nH/mm) and capacitances may be obtained by using low Z_0 microstrip lines (for $Z_0 = 20\ \Omega$, $C_\ell = \frac{1}{v_p Z_0} = 0.283 \frac{pF}{mm}$).
Alternatively the capacitances may be obtained by using open circuited stub lines.

$$Y = j\omega C = j Y_{0,s} \tan\left(\frac{\omega \ell}{v_p}\right)$$

As given in the writeup for Lab. 4 for filter # 2, N=7 filter may also be used.
From Table 8.4 (p.396) for N=7 filter (0.5 dB ripple)

$g_1 = g_7 = \frac{\omega L}{Z_0} = 1.7372$; $g_2 = g_6 = \frac{\omega C}{Y_0} = 1.2583$; $g_3 = g_5 = \frac{\omega L}{Z_0} = 2.6381$

$g_4 = \frac{\omega C}{Y_0} = 1.3444$

Design of the filter circuit using Microstrip Lines

For any transmission line, from Eqns. 2.13, 2.16 p. 52

Inductance per unit length $L_\ell = \dfrac{Z_0}{v_p}$

Capacitance per unit length $C_\ell = \dfrac{1}{v_p Z_0}$

For inductances use a high Z_0 line, say $Z_0' = 120\,\Omega$

$v_p = \dfrac{c}{\sqrt{\epsilon_{eff}}}$ say $\epsilon_{eff} = 1.7$

$$L_\ell = \frac{Z_0' \sqrt{\epsilon_{eff}}}{c} = \frac{120\sqrt{1.7}}{3\times 10^8} = 521.5\ \frac{nH}{m} \Rightarrow 0.521\ \frac{nH}{mm}$$

$\left(0.651\ \frac{nH}{mm} \text{ for } Z_{0h} = 150\,\Omega\right)$

For capacitance use either a low Z_0 line or better an open circuited stub. $C_\ell = \dfrac{\sqrt{\epsilon_{eff}}}{c Z_0} = 0.283\ pF/mm$ for $Z_0 = 20\,\Omega$ line

For an open circuited stub, the equivalent capacitance C_{eq} is given by:

$$Y = j\, Y_{0,s} \tan(\beta \ell) \Rightarrow j\omega C_{eq}$$

For the shunt arms 1, 2 use open circuited stubs of lengths $\lambda_g/4$ at 3.75 GHz and 3.075 GHz, respectively.

$$Y_{stub} = j\, Y_{0s} \tan \pi/2 = \infty$$

$$Z_{stub} = 0$$

The shunt arms 1 and 2 amount to an impedance of 0 or short circuit at 3.75 and 3.075 GHz.

A high pass filter with $f_c = 4.0\,GHz$

From Table 8.2 p. 387

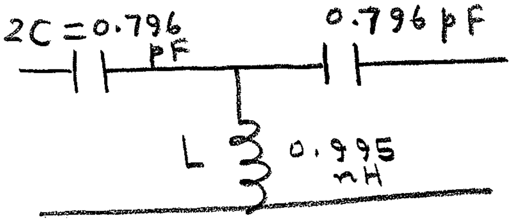
$$R_0 = \sqrt{L/C} = 50\,\Omega$$

$$w_c = \frac{1}{2\sqrt{LC}}$$

$$L = \frac{R_0}{2w_c} = \frac{50}{2\times 2\pi\times 4\times 10^{9}} = 0.995\ nH$$

$$C = \frac{1}{2w_cR_0} = \frac{1}{2\times 2\pi\times 4\times 10^{9}\times 50} = 0.398\ pF$$

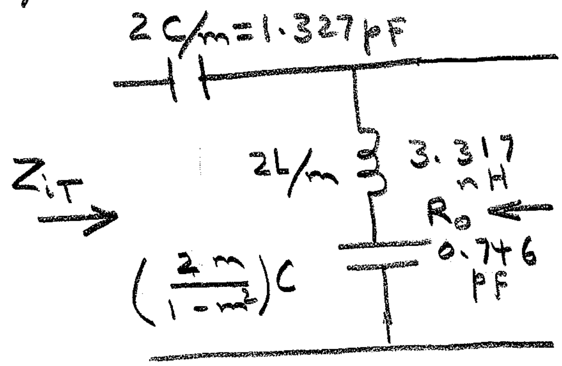
Constant-k T Section



m = 0.6 filter for matching

$$f_\infty = f_c\sqrt{1-m^2} = 4\times 0.8 = 3.2\,GHz$$

1/2 π m-derived filter

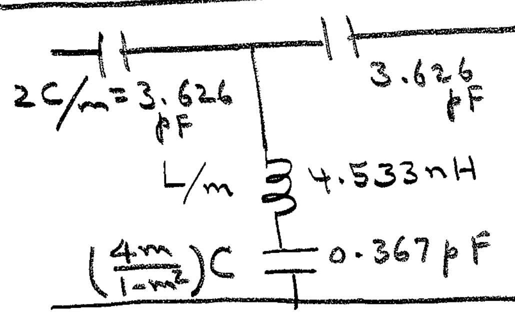


Note that the resonant frequency of the shunt arm is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\times\sqrt{3.317\times10^{-9}\times 0.746\times10^{-12}}} = 3.2\times10^{9} \rightarrow 3.2\,GHz$$

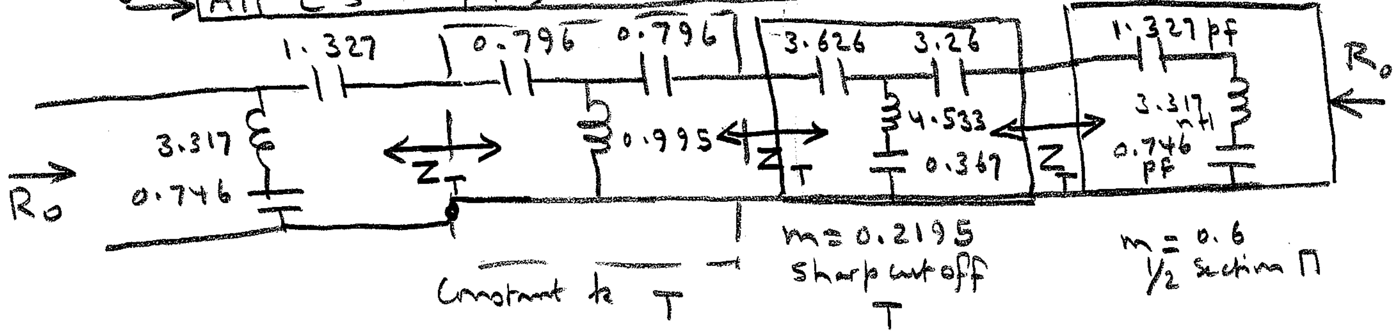
The shunt arm can be realized as an open-circuited shunt stub of length $\lambda g/4$ at 3.2 GHz

m = 0.2195 sharp cut off filter, full section T



We can combine the various stages to create a four-stage composite filter of configuration similar to Fig. 8.18 p. 387

$\boxed{\text{All C's in pF; All L's in nH}}$



pp. 398-401 Text

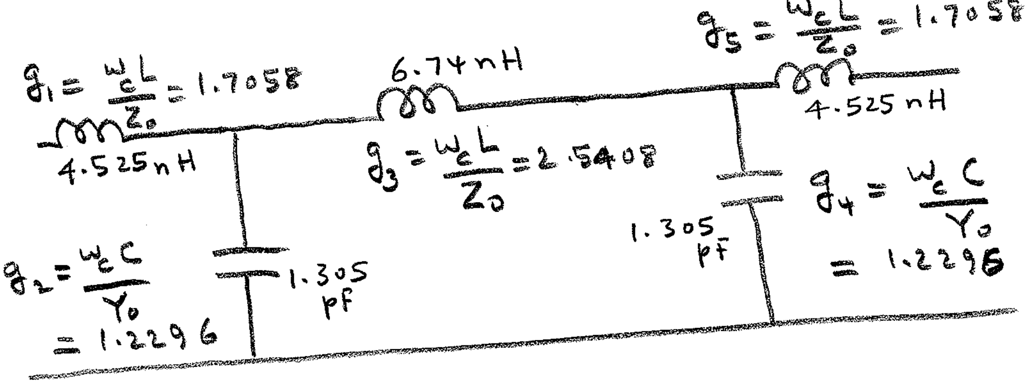
8.4 Transformation of Low-Pass to High-Pass Filters

For maximally Flat or Equal-ripple filters

See also Table 8.6

Low-Pass		High-Pass	
$g_K = \dfrac{\omega_c L}{Z_0}$	L	C	$\dfrac{\omega_c' C}{Y_0} = \dfrac{1}{g_K}$
$g_K = \dfrac{\omega_c C}{Y_0}$	C	L	$\dfrac{\omega_c' L}{Z_0} = \dfrac{1}{g_K}$

For the low-pass filter on p. 411 (0.5 dB N = 5 Equal-ripple filter)



Transformed High-Pass Filter