

Conceptually Understanding Linear Functions

Teacher Notes

This packet is conceptually difficult, but this is the way we wish all our incoming electrical engineering students could mathematically think. This packet is best for an honors classroom.

Middle School:

Standard 6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.

Standard 7.RP.2 Recognize and represent proportional relationships between quantities.

Part 2: Standard MP.6 Attend to precision.

Mathematics I:

Standard A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's Law $V = IR$ to highlight resistance R .*

Part 2: SI.MP.6 Attend to precision. **Standard N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Supplies needed:

2 AA batteries

Battery holders

Resistor Sheet

Current meter

Alligator clip wires

Overview:

Electrical devices work through the flow of electrons, which are charged particles. For an electrical circuit to work, it must have something that provides power (like a battery), something that consumes power (like a resistor), and wires must connect everything in a circle, so the electrons can flow. The battery has a given *voltage*, which provides the “umph” needed to get the electrons flowing. We call the flow of electrons *current*.

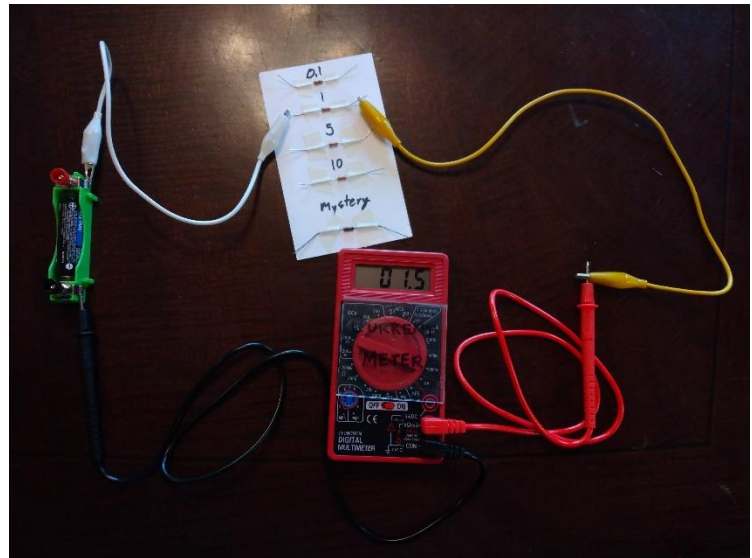
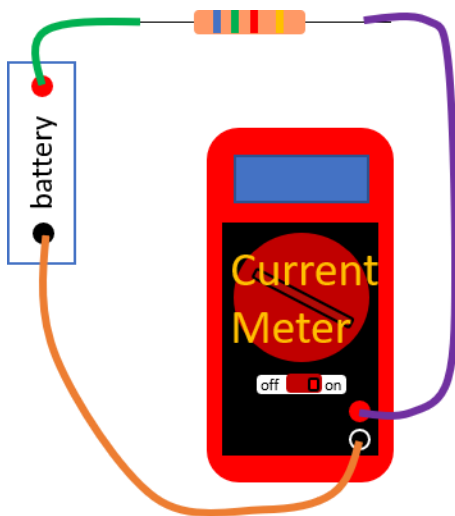
How much current flows depends on the voltage of the battery and the resistance of the resistor. According to the famous electrical engineering equation Ohm's Law,

Voltage = Current · Resistance

$$V = I \cdot R$$

This is a linear equation. That means that for a given resistor, the ratio of the voltage and current will remain constant. More generally, if any one of the variables in the equation is held constant, the other two will be in a fixed proportion or ratio.

- Build the following circuit by connecting a AA battery, the 1 resistor, and the current meter in a loop. Record the current. A single battery has a voltage of approximately 1.5 volts. Use this information and Ohm's Law to figure out what the resistance of the resistor is.



Sample values:

(given) **Voltage = 1.5**

(measure) **Current = 1.6** (example measurement)

[Answers should be between 1.4-1.6...if not...troubleshoot!]

(calculate) **Resistance = $\frac{V}{I} = \frac{1.5}{1.6} = 0.94$**

Common issues to look for when troubleshooting:

- Mixed up current and voltage meter
- Did not turn on current or voltage meter
- Missing connections (forgot to connect everything in a circle)
- Loose connections (check alligator clips and check red and black cables firmly pushed into holes on meters)
- Batteries not all in same direction
- Batteries dead

- How close is this to the labeled resistance? Why is it not exact?

It is 0.06 less than the labeled value. The battery might not be exactly 1.5 (battery voltages can range from 1.35 for an almost dead battery to 1.6 for a brand new battery). The resistor might also not be exactly 1 (resistors have a variation in value of 5%, which is called the resistor tolerance). For students who have larger resistance values than 1: Bad connections can also add resistance to the circuit.

Now we will make changes to the original circuit. Use the equation Ohm's Law to *predict* what will happen in the following situations.

3. By putting two batteries in a row, we can double the voltage. If we use the same resistor, how will the current change?
- a. First determine which variable is fixed (constant), which variable is the independent variable, and which variable is the dependent variable. *Hint: Remember that the independent variable is the one we are controlling.*

(circle one for each)

Constant Voltage or Current or **Resistance**
Independent Variable **Voltage** or Current or Resistance
Dependent Variable Voltage or **Current** or Resistance

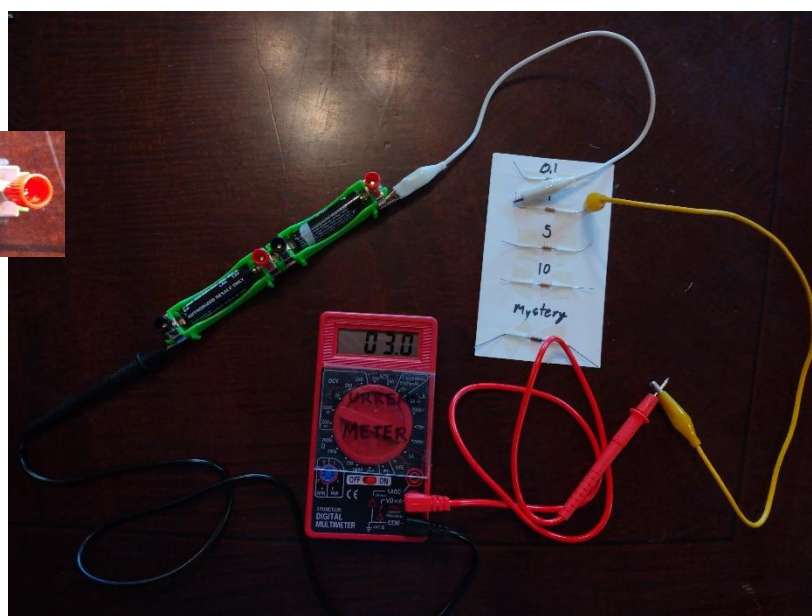
- b. Now predict how the current will change if the voltage is doubled using Ohm's Law: $Voltage = Current \cdot Resistance$.
Hint: It may be helpful to rearrange the equation so the dependent variable is by itself.

One way to approach this problem is to think about what needs to be done to keep both sides of the equation equal. If the independent and dependent variables are on opposite sides of the equation, then if one is doubled, the other one has to double to keep the sides of the equation equal.

Another way to approach this problem is to have them figure out if the two variables are proportional or inversely proportional. In this case the variables are proportional.

If the voltage is doubled, then the current will STAY THE SAME / **DOUBLE** / HALF (circle one).

- c. Build the circuit and confirm your prediction. Make sure you put your batteries with the red sides facing in the same direction.



Current = 3.0 (example measurement)

The current **DOUBLED** / HALVED / STAYED THE SAME (circle one)
 compared to the current measured in 1.

4. By putting two resistors in a row, we can double the resistance. If we use the same single battery as the original situation, then how will current change?
- a. What are the constant, independent variable and dependent variable this time?

(circle one for each)

Constant Voltage or Current or Resistance

Independent Variable Voltage or Current or Resistance

Dependent Variable Voltage or Current or Resistance

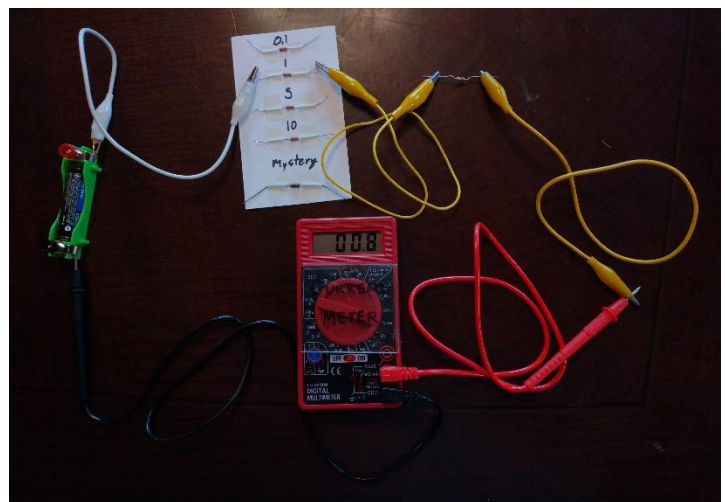
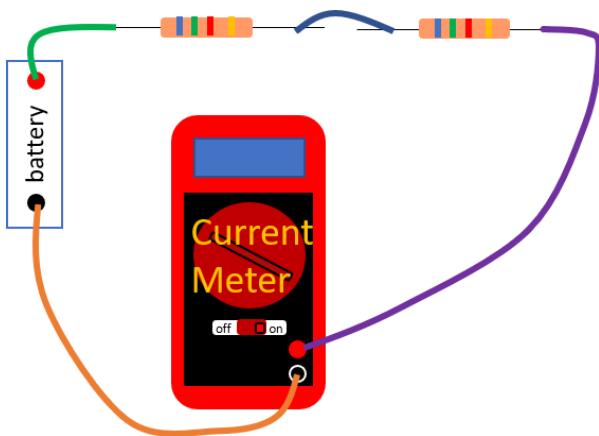
- b. Predict how the current will change if the resistance is doubled using Ohm's Law: Voltage = Current · Resistance
Hint: It may be helpful to figure out if the dependent and independent variable are proportional or inversely proportional.

This situation gets a little more tricky when the dependent and independent variables are on the same side of the equation. Again, we can think about keeping both sides of the equation equal under changes. If we double the independent variable, what must be done to the dependent variable to keep that side of the equation equal to the constant on the other side of the equation? Hint: The operation must cancel out the doubling.

Another way to approach this problem is to have them figure out if the two variables are proportional or inversely proportional. In this case, they are inversely proportional.

If the resistance is doubled, then the current will STAY THE SAME / DOUBLE / HALF (circle one)

- c. Build the circuit and confirm your prediction. There is a second resistor of resistance 1 in the plastic baggie.



Current = 0.8 (example measurement)

The current DOUBLED / HALVED / STAYED THE SAME (circle one)

compared to the current measured in 1.

5. (Bonus) What happens if you make both changes at once? (Add two batteries in a row and two resistors in a row)
- Predict how the current will change.

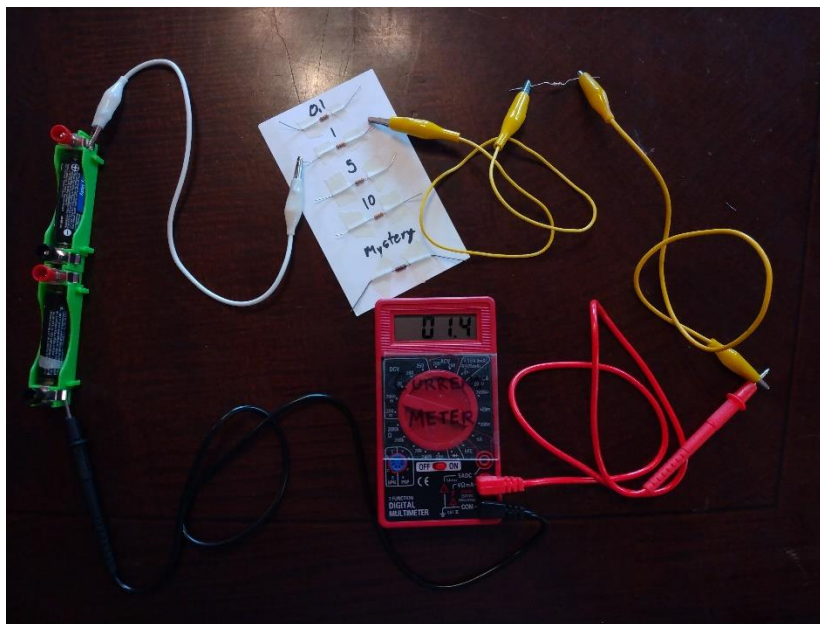
If both the voltage and resistance are doubled, then the current will

STAY THE SAME / DOUBLE / HALF (circle one)

compared to the current measured in 1.

This example will hopefully make a little more clear why looking at balancing both sides of the equation is a helpful approach as you get into more advanced engineering. You may make multiple changes or have a more complicated, compound equation (with exponents, etc.) that is hard to rearrange and want to know how changes to input variables affects the output you care about.

- Confirm your prediction by building the circuit.



Current = 1.4 (example measurement)

The current DOUBLED / HALVED / **STAYED THE SAME** (circle one)

compared to the current measured in 1.