

Appendix, Calculations

Series RLC Circuit

$$R_T := R + R_s + R_{sub}$$

For transient analysis, use the LaPlace s instead of ω for the impedances.
Remember that the LaPlace $s = \alpha + j\omega$

$$\text{Transfer function: } H(s) = \frac{R}{Ls + R_T + \frac{1}{C \cdot s}} = R \cdot \frac{s}{L \cdot s^2 + R_T \cdot s + \frac{1}{C}} = \frac{R}{L} \cdot \frac{s}{s^2 + \frac{R_T}{L} \cdot s + \frac{1}{L \cdot C}}$$

voltage divider

If you take the denominator of the transfer function and set it equal to zero, you get the characteristic equation:

$$\text{Characteristic equation: } 0 = s^2 + \frac{R_T}{L} \cdot s + \frac{1}{L \cdot C}$$

Solve the characteristic equation for s values, using the quadratic equation:

$$s_1 := \frac{-\frac{R_T}{L} + \sqrt{\left(\frac{R_T}{L}\right)^2 - \frac{4}{L \cdot C}}}{2}$$

$$s_2 := \frac{-\frac{R_T}{L} - \sqrt{\left(\frac{R_T}{L}\right)^2 - \frac{4}{L \cdot C}}}{2}$$

$$s_1 = -2.679 \cdot 10^4 + 5.97 \cdot 10^5 j \quad \cdot \frac{1}{\text{sec}} \quad s_2 = -2.679 \cdot 10^4 - 5.97 \cdot 10^5 j \quad \cdot \frac{1}{\text{sec}}$$

$$s = \alpha + j\omega, \text{ so: } \alpha := \frac{-R_T}{2 \cdot L} \quad \alpha = -26786 \cdot \frac{1}{\text{sec}} \quad \text{and: } \omega := \frac{1}{2} \sqrt{\frac{4}{L \cdot C} - \left(\frac{R_T}{L}\right)^2} \quad \omega = 5.97 \cdot 10^5 \cdot \frac{1}{\text{sec}}$$

$e^{\alpha t}$ is a decaying exponential

$$\text{The time constant is: } \tau := -\frac{1}{\alpha} \quad \tau = 37.3 \cdot \mu\text{s}$$

$$f := \frac{\omega}{2 \cdot \pi} \quad f = 95 \cdot \text{kHz}$$

Compare these to what you measured.

Critical Damping happens when the part of s under the radical is 0:

$$\left(\frac{R_T}{L}\right)^2 = \frac{4}{L \cdot C}$$

$$R_T = \sqrt{\frac{L \cdot 4}{C}} = 3347 \cdot \Omega$$

Parallel RLC Circuit

$$\text{Impedance of } C, L, \text{ \& } R_L: Z(s) = \frac{1}{C \cdot s + \frac{1}{L \cdot s + R_L}}$$

Transfer function:

$$H(s) = \frac{Z(s)}{Z(s) + R} = \frac{1}{1 + \frac{R}{Z(s)}} = \frac{1}{1 + R \cdot \left(C \cdot s + \frac{1}{L \cdot s + R_L}\right)}$$

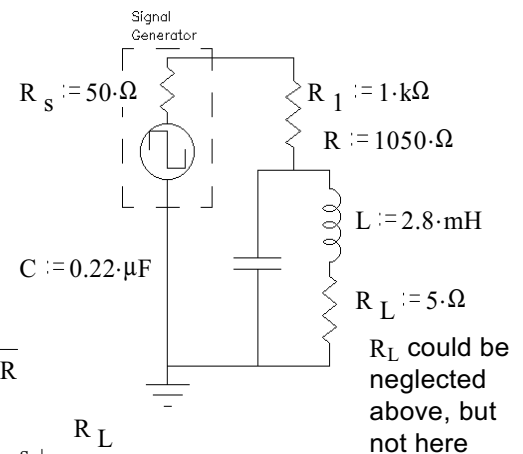
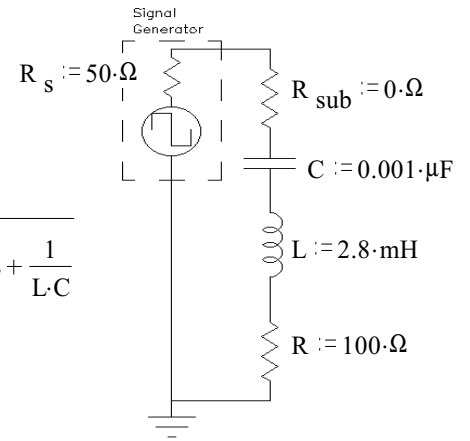
$$= \frac{1}{1 + R \cdot C \cdot s + \frac{R}{L \cdot s + R_L}} \cdot \left(\frac{L \cdot s + R_L}{L \cdot s + R_L}\right) = \frac{L \cdot s + R_L}{L \cdot s + R_L + R \cdot C \cdot s \cdot (L \cdot s + R_L) + R}$$

$$= \frac{L \cdot s + R_L}{R \cdot C \cdot L \cdot s^2 + (L + R \cdot C \cdot R_L) \cdot s + (R_L + R)} \cdot \left(\frac{1}{R \cdot C \cdot L}\right) = \frac{\frac{1}{R \cdot C} \cdot s + \frac{R_L}{R \cdot C \cdot L}}{s^2 + \left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right) \cdot s + \left(\frac{R_L}{R \cdot C \cdot L} + \frac{1}{C \cdot L}\right)}$$

$$\text{characteristic equation: } 0 = \left[s^2 + \left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right) \cdot s + \left(\frac{R_L}{R \cdot C \cdot L} + \frac{1}{C \cdot L}\right) \right]$$

$$\text{Find solutions to the characteristic eq. as above: } = -\frac{1}{2} \cdot \left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right) \pm \frac{1}{2} \cdot \sqrt{\left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right)^2 - 4 \cdot \left(\frac{R_L}{R \cdot C \cdot L} + \frac{1}{C \cdot L}\right)}$$

$$\alpha := -\frac{1}{2} \cdot \left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right) \quad \tau = -\frac{1}{\alpha} = 0.327 \cdot \text{ms} \quad \omega := \frac{1}{2} \cdot \sqrt{\left(\frac{1}{R \cdot C} + \frac{R_L}{L}\right)^2 - 4 \cdot \left(\frac{R_L}{R \cdot C \cdot L} + \frac{1}{C \cdot L}\right)} \quad f = \frac{\omega}{2 \cdot \pi} = 6.41 \cdot \text{kHz}$$



R_L could be neglected above, but not here