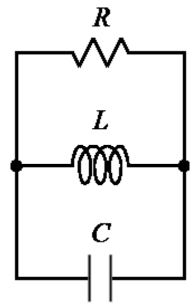
Homework 10

1.



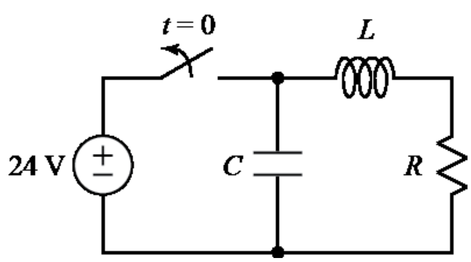
$R = 0.5\ \Omega$ $L = 1.5\ \mu H$ $C = 1.5\ \mu F$

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Is the circuit over-damped, critically-damped, or under-damped? Explain.

c) If the L and C values in the circuit are decreased by a factor of two, (and R remains the same), will the circuit be over-damped, critically-damped, or under-damped? Justify your answer with calculations.

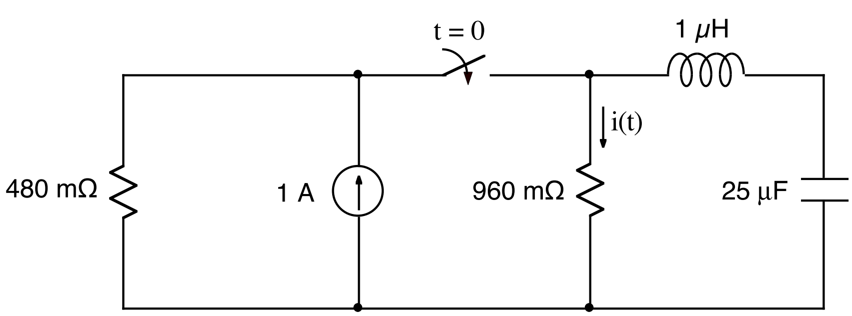
2.



$C = 1$ mF $L = 25$ mH

A relay is driven by a 24 V power supply, as shown above. Power is turned off at $t = 0$. The current, $i(t)$, for $t > 0$ has two terms that decay exponentially without oscillation. One term dies out quickly, and the other term dies out with a time constant of $\tau = 10$ ms, as in $e^{-t/10\text{ms}}$. Given the time constant and the information in the diagram above, find the value of R.

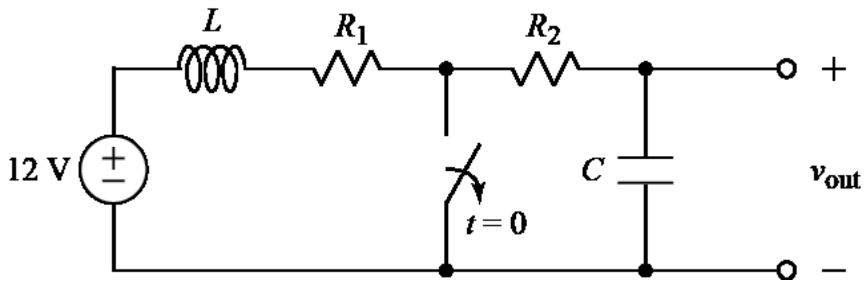
3.



After being open for a long time, the switch closes at $t = 0$.

Find $i(t)$ for $t > 0$.

4.



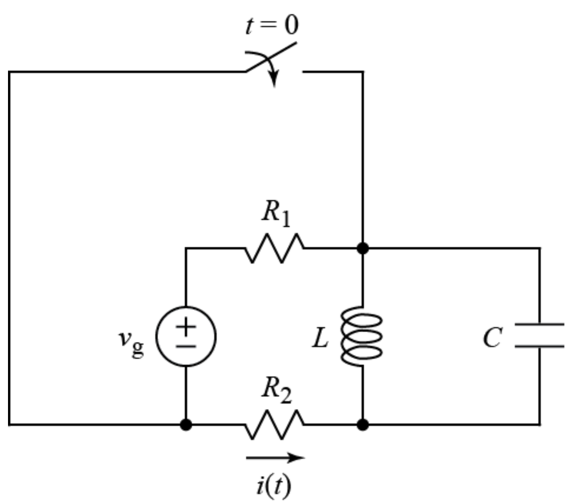
A 12 V power supply drives a long wire, (modeled as L and R_1), followed by a short wire, R_2, and a smoothing capacitor, C. There is a safety switch, located before the smoothing capacitor, to turn off the output at the remote end. The switch is closed for a long time before opening at $t = 0$.

$L = 2\ \mu\text{H} \qquad R_1 = 2.0\ \Omega \qquad R_2 = 0.1\ \Omega \qquad C = 200\ \mu\text{F}$

a) Find the characteristic roots, s_1 and s_2, for the above circuit.

b) Find v_{out} for $t > 0$.

5.



After being open for a long time, the switch closes at $t = 0$.

a) Give expressions for the following in terms of no more than v_g, R_1, R_2 L, and C:

$$i(t = 0^+) \qquad \text{and} \qquad \left.\frac{di(t)}{dt}\right|_{t=0^+}$$

b) Find the numerical value of R_2 given the following information:

$R_1 = 150\ \Omega \qquad L = 40\ \text{mH} \qquad C = 3.2\ \mu\text{F}$

$\alpha = 1250\ \text{r/s} \qquad \omega_d = 2500\ \text{r/s}$

Answers:

1.a) Duplicate roots = -2/3 Mr/s. c) critically damped

2. R = 12.5 Ω

3. $i(t) = -\frac{8}{9} e^{-160\text{k}t/s} \sin(120\text{k}\, t) + \frac{1}{3}$ A

4.a) Partial answer: $s_1 \approx -2.4\,\text{kr/s}$ overdamped.

b) $v_{\text{out}}(t > 0) \approx -12.06 e^{-2.4\text{k/s}\, t} + 0.06 e^{-1.05\text{M/s}\, t} + 12\,\text{V}$ Find a way to handle approximations.

5.a) Partial answer: $\left.\frac{di(t)}{dt}\right|_{t=0^+} = \frac{1}{R_2} \frac{i_C(t = 0^+)}{C} = -\frac{v_g}{R_2 C (R_1 + R_2)}$. b) R_2 = 125 Ω.