1. Sketch each of the following functions and then express each of them as a summation of products of common functions and step functions.

a) $f(t) = \begin{cases} 0 & t \leq 1 \\ 2(t-1)^2 & 1 \leq t \leq 2 \\ 2 & 2 \leq t \leq 5 \\ 2e^{t-5} & 5 \leq t \leq 8 \\ 0 & 8 < t \end{cases}$

b) $f(t) = \begin{cases} 0 & t < 1 \\ 4e^{-t/3}\cos(\pi t) & 1 \leq t \leq 4 \\ 0 & 4 < t \end{cases}$

2. Compute the Laplace transform of the following functions by calculating the integral expression for the Laplace transform (step-by-step by hand):

a) $f(t) = u(t) - u(t-1)$ where $u(t)$ is the unit step function: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t \end{cases}$

b) $f(t) = \begin{cases} 0 & t < 0 \\ e^{-2t} & 0 \leq t \end{cases}$

c) $f(t) = \begin{cases} 0 & t < 4 \\ e^{-2(t-4)} & 4 \leq t \end{cases}$

3. Using algebraic simplification and a table of Laplace transform pairs, find the Laplace transform of each of the following functions:

a) $f(t) = 3e^{-(t-1)}$

b) $f(t) = \cos(2\pi t + 30°)$

c) $f(t) = 4 + t$

4.   Using algebraic simplification and a table of Laplace transform pairs, find the inverse Laplace transform of each of the following functions:

a) $F(s) = 3e^{-(s-1)}$

b) $F(s) = \dfrac{1}{s} + \dfrac{3}{s^2 + 4}$

c) $F(s) = \dfrac{s+5}{s^2 + 2s + 5}$

5.   Find the values of the following expressions:

a) $\displaystyle\int_{-1}^{1} \delta(t)\,dt$

b) $\displaystyle\int_{-\infty}^{\infty} t^2 \delta(t-2)\,dt$

c) $f(t) = \begin{cases} 0 & t \le -1 \\ \displaystyle\int_{-1}^{t} \tau^2 \delta(\tau - 2)\,d\tau & t > -1 \end{cases}$

d) $\displaystyle\int_{-\infty}^{\infty} \delta(t/2)\,dt$

e) $\displaystyle\int_{-\infty}^{t} \left(\int_{-\infty}^{T} 2\delta(\tau)\,d\tau \right) dT$

Answers:

1.a) $f(t) = 2(t-1)^2[u(t-1)-u(t-2)]$
$+2[u(t-2)-u(t-5)]$
$+2e^{t-5}[u(t-5)-u(t-8)]$

or

$f(t) = 2(t-1)^2 u(t-1) + [2t(2-t)]u(t-2)$
$+\left[2\{e^{t-5}-1\}\right]u(t-5) - 2e^{t-5}u(t-8)$

2.a) $\dfrac{1}{s} - \dfrac{e^{-s}}{s}$ (you must show the integration steps)   c) $F(s) = \dfrac{e^8}{s+2}$

3.a) Hint: $3e^{-(t-1)} = \dfrac{3}{e} \cdot e^{-t}$ b) Hint: use identity for $\cos(A+B)$ c) $F(s) = \dfrac{4}{s} + \dfrac{1}{s^2}$

4.a) $\mathcal{L}^{-1}\{3e^{-(s-1)}\} = 3e\mathcal{L}^{-1}\{e^{-s}\} = 3e\delta(t-1)$

b) Hint: $3 = \dfrac{3}{2} \cdot 2$

c) Hint: $s+5 = (s+1)+2(2)$ split the expression into two fractions

5.a) 1

b) 4

c) Hint: $\displaystyle\int_{-\infty}^{\infty} f(\tau)\delta(\tau-a)d\tau = f(a)$

d) Hint: use a change of variables to $\tau = \dfrac{t}{2}$

e) Hints: $\displaystyle\int_{-\infty}^{t} \delta(t)dt = u(t)$ and $\displaystyle\int_{-\infty}^{t} f(t)dt =$ Area under $f(t)$ to the left of t