# Homework 12

1. Find the Laplace transforms of the following waveforms:

   a) $(t-a)\cos(t-b)u(t-a)$ where $a>0$

   b) $\dfrac{d}{dt}\left[t\sin(\omega t)+t^2\right]$

   c) $f(t)=t\cos(\omega t)e^{-at}$

   d) $t\int_0^t te^{-at}\,dt$

2. Find the Laplace transform of the following waveform:

   $t\sin(\omega t)\cos(\omega t)$

3. Find the inverse Laplace transform for each of the following expressions:

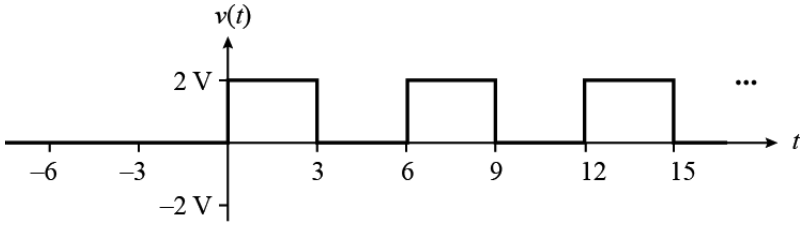
   a) $F(s)=\dfrac{5s+18}{s^2+6s}$

   b) $F(s)=\dfrac{7s+70}{s^2+8s+25}$

   c) $F(s)=\dfrac{-3s^2+99s-1200}{s^3+11s^2+100s+1100}$

   d) $F(s)=\dfrac{15s^2+186s+624}{s^3+18s^2+112s+160}$

4. Find the Laplace transform, if possible, of the following square wave:




5. Find the inverse Laplace transform of $\dfrac{24s}{(s+5)^4}$. Note: $\mathcal{L}\{t^n e^{-at}\}=\dfrac{n!}{(s+a)^{n+1}}$

Answers:

1.a) $\mathcal{L}\{(t-a)\cos(t-b)u(t-a)\}$

$$= e^{-as}\left\{\cos(a-b)\left[\frac{s^2-1}{(s^2+1)^2}\right]-\sin(a-b)\left[\frac{2s}{(s^2+1)^2}\right]\right\}$$

b) $\mathcal{L}\left\{\frac{d}{dt}\left[t\sin(\omega t)+t^2\right]\right\}$

$$= \frac{\omega 2s^2}{(s^2+\omega^2)^2}+\frac{2}{s^2}$$

c) $\mathcal{L}\left\{t\cos(\omega t)e^{-at}\right\} = \frac{(s+a)^2-\omega^2}{[(s+a)^2+\omega^2]^2}$

d) $\mathcal{L}\left\{t\int_0^t te^{-at}\,dt\right\} = \frac{3s+a}{s^2(s+a)^3}$

2. Hint: $\sin(A)\cos(A) = \frac{1}{2}\sin(2A)$

3.a) $(3+2e^{-6t})u(t)$

b) $[7e^{-4t}\cos(3t)+14e^{-4t}\sin(3t)]u(t)$

c) Hint: 11 is one root of the denominator. Ans: $[-12e^{-11t}+9\cos(10t)]u(t)$

d) $[6e^{-2t}+9e^{-8t}\cos(4t)]u(t)$

4. Hints: Laplace transform of sum = sum of Laplace transforms, and $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$.

5. $[-20t^3e^{-5t}+12t^2e^{-5t}]u(t)$