HOMEWORK #12 prob 1b solution

Ex: Find the Laplace transform of the following waveform:

$$\frac{d}{dt}\left[t\sin(\omega t)+t^2\right]$$

Sol'n: Use the derivative identity and the multiplication by t identity.

$$\mathcal{L}\left\{\frac{dv(t)}{dt}\right\}=sV(s)-v(t=0^-) \quad \text{and} \quad \mathcal{L}\{tv(t)\}=-\frac{dV(s)}{ds}$$

We start on the inside and work our way out.

$$\mathcal{L}\{t\sin(\omega t)\}=-\frac{d}{ds}\mathcal{L}\{\sin(\omega t)\}=-\frac{d}{ds}\frac{\omega}{s^2+\omega^2}=\frac{\omega 2s}{(s^2+\omega^2)^2}$$

and

$$\mathcal{L}\{t^2\}=-\frac{d}{ds}\mathcal{L}\{t\}=-\frac{d}{ds}\frac{1}{s^2}=\frac{2}{s^3}$$

Now apply the derivative identity:

$$\mathcal{L}\left\{\frac{d}{dt}\left[t\sin(\omega t)+t^2\right]\right\}=s\mathcal{L}\{t\sin(\omega t)+t^2\}-\left[t\sin(\omega t)+t^2\right]\Big|_{t=0^-}$$

$$=s\frac{\omega 2s}{(s^2+\omega^2)^2}+s\frac{2}{s^3}$$

$$=\frac{\omega 2s^2}{(s^2+\omega^2)^2}+\frac{2}{s^2}$$

Note that the evaluation of the function at $t = 0^-$ gives a value of zero.