Ex: Find the Laplace transform of the following waveform:

$$t\sin(\omega t)\cos(\omega t)$$

SOL'N: We apply the trigonometric identity for the product of sine and cosine:

$$\sin(A)\cos(A) = \frac{1}{2}\sin(2A)$$

Thus, we have the following result:

$$\mathcal{L}\{\sin(\omega t)\cos(\omega t)\} = \frac{1}{2}\mathcal{L}\{\sin(2\omega t)\} = \frac{1}{2}\frac{2\omega}{s^2 + (2\omega)^2} = \frac{\omega}{s^2 + (2\omega)^2}$$

Now we apply the identity for multiplication by t:

$$\mathcal{L}\{tv(t)\} = -\frac{dV(s)}{ds}$$

We obtain the following result:

$$\mathcal{L}\{t\sin(\omega t)\cos(\omega t)\} = -\frac{d}{ds}\frac{\omega}{s^2 + (2\omega)^2} = -\frac{\omega(-1)2s}{\left(s^2 + (2\omega)^2\right)^2}$$

$$= \frac{2\omega s}{\left(s^2 + (2\omega)^2\right)^2}$$