Ex: Find the inverse Laplace transform for the following expression:

$$F(s)=\frac{15s^2+186s+624}{s^3+18s^2+112s+160}$$

Sol'n: Using a calculator or other root solver, we find one root term of the denominator is $s + 2$.

$$F(s)=\frac{15s^2+186s+624}{(s+2)(s^2+16s+80)}$$

The other two roots are complex, and we use an expansion in terms of a decaying cosine and sine:

$$F(s)=\frac{A}{(s+2)}+\frac{B(s+8)+C(4)}{(s+8)^2+4^2}$$

We find A by the pole cover-up method.

$$A=F(s)(s+2)\Big|_{s=-2}=\left.\frac{15s^2+186s+624}{(s+8)^2+4^2}\right|_{s=-2}=\frac{15(4)-186(2)+624}{(-2+8)^2+4^2}$$

$$=\frac{312}{52}=6$$

Using this value of A, we put everything over a common denominator.

$$F(s)=\frac{6}{(s+2)}+\frac{B(s+8)+C(4)}{(s+8)^2+4^2}$$

$$=\frac{6[(s+8)^2+4^2]+(s+2)[B(s+8)+C(4)]}{(s+2)[(s+8)^2+4^2]}$$

Now we match the numerator to the original $F(s)$.

$$6[(s+8)^2+4^2]+(s+2)[B(s+8)+C(4)]$$
$$=15s^2+186s+624$$

or

$$6s^2+96s+480+Bs^2+2Bs+8Bs+4Cs+16B+8C$$
$$=15s^2+186s+624$$

Matching the coefficients of powers of s we have

$$s^2: \; 6 + B = 15$$
$$s: \quad 96 + 2B + 8B + 4C = 186$$
$$1: \quad 480 + 16B + 8C = 624$$

or

$$B = 9$$
$$C = 0$$

Thus,

$$F(s) = \frac{6}{(s+2)} + 9\frac{s+8}{(s+8)^2 + 4^2}$$

and

$$f(t) = \left[6e^{-2t} + 9e^{-8t}\cos(4t)\right]u(t)$$