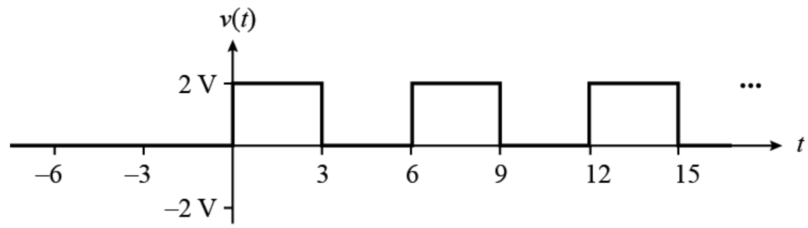
EX: Find the Laplace transform, if possible, of the following square wave:



SOL'N: We write the square wave as a summation of step functions.

$$v(t) = 2\,\mathrm{V} \cdot \sum_{k=0}^{\infty} u(t-6k) - u(t-3-6k)$$

We take the Laplace transform inside the summation since the Laplace transform of a sum is the sum of the Laplace transforms.

$$V(s) = 2\,\mathrm{V} \cdot \sum_{k=0}^{\infty} \left(\mathcal{L}\{u(t-6k)\} - \mathcal{L}\{u(t-3-6k)\} \right)$$

Now we apply the delay identity to each term.

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{f(t)\}$$

Here, our function is $f(t-a) = 1$.

$$V(s) = 2\,\mathrm{V} \cdot \sum_{k=0}^{\infty} \left(e^{-6ks}\mathcal{L}\{1\} - e^{-(6k+3)s}\mathcal{L}\{1\} \right)$$

or

$$V(s) = 2\,\mathrm{V} \cdot \sum_{k=0}^{\infty} \left(\frac{e^{-6ks}}{s} - \frac{e^{-(6k+3)s}}{s} \right)$$

or if we factor out the e^{-6ks}

$$V(s) = 2\,\mathrm{V} \cdot \sum_{k=0}^{\infty} \left(\frac{1-e^{-3s}}{s} e^{-6ks} \right).$$

We can move terms not containing k outside the summation.

$$V(s) = 2\,\mathrm{V}\left(\frac{1-e^{-3s}}{s} \right) \cdot \sum_{k=0}^{\infty} e^{-6ks}$$

Now we apply the formula for summation of geometric series (knowing that it only applies if $|x| < 1$.

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

Our x is e^{-6s}:

$$V(s) = 2\,\mathrm{V}\left(\frac{1-e^{-3s}}{s}\right)\cdot\frac{1}{1-e^{-6s}}$$