EX: Find $v(t)$ if $V(s) = 2 + \dfrac{s^2 + s + 4}{s(s^2 + 4)}$.

SOL'N:

We know $\mathcal{L}^{-1}\{2\} = 2\delta(t)$, so we focus on the inverse transform of $\dfrac{s^2+s+4}{s(s^2+4)}$.

$$\frac{s^2+s+4}{s(s^2+4)} = \frac{A}{s} + \frac{Bs + C(2)}{(s^2+\underset{\omega}{2^2})}$$

Note: the second term is written as the form for a cosine plus sine (in t-domain):

$$B\frac{s}{s^2+\omega^2} + C\frac{\omega}{s^2+\omega^2} \xrightarrow{\mathcal{L}^{-1}} B\cos\omega t + C\sin\omega t$$

Using the pole cover-up method, we find A:

$$A = \cancel{s}\left(\frac{s^2+s+4}{\cancel{s}(s^2+4)}\right)\Bigg|_{s=0} = \frac{4}{4} = 1$$

Subtracting the $\frac{1}{s}$ term leaves $\dfrac{Bs+C(2)}{s^2+2^2}$:

$$\frac{s^2+s+4}{s(s^2+4)} - \frac{1}{s} = \frac{s^2+s+4}{s(s^2+4)} - \frac{(s^2+4)}{s(s^2+4)} = \frac{\cancel{s}1}{\cancel{s}(s^2+4)}$$

So we have $\dfrac{1}{s^2+4} = \dfrac{\frac{1}{2}(2)}{s^2+4}$, so $B=0$, $C = 1/2$.

$$\mathcal{L}^{-1}\left\{\frac{1}{2}\cdot\frac{2}{s^2+4}\right\} = \frac{1}{2}\sin(2t)\,u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$$

Thus,

$$v(t) = 2\delta(t) + \left[1 + \frac{1}{2}\sin(2t)\right]u(t)$$