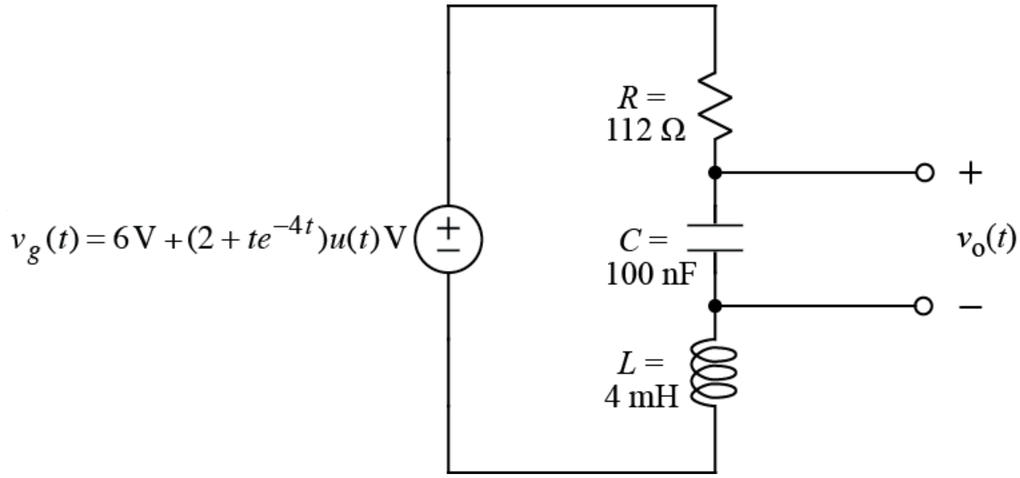
HOMEWORK #14 probs 1 and 2 solution

Ex:



Note: The 6 V in the $v_g(t)$ source is always on.

a) Write the Laplace transform, $V_g(s)$, of $v_g(t)$.

b) Draw the s-domain equivalent circuit, including source $V_g(s)$, components, initial conditions for C's, and terminals for $V_o(s)$.

c) Write an expression for $V_o(s)$.

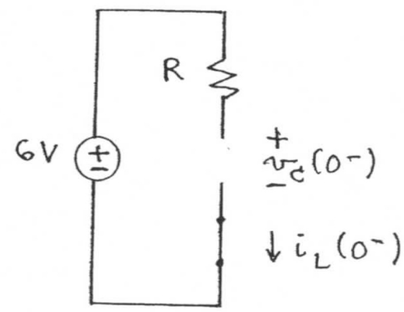
d) Apply the final value theorem to find $\lim_{t\to\infty} v_o(t)$.

SOL'N: a) $\mathcal{L}\{6\text{V} + (2 + te^{-4t})u(t)\text{V}\}$

$$= \frac{6}{s} + \frac{2}{s} + \frac{1}{(s+4)^2} = V_g(s)$$

b) We find the initial conditions for L and C at $t = 0^-$. Because we have 6 VDC input before $t = 0$, we expect the circuit to reach equilibrium, meaning we may treat the L as a wire and the C as an open.

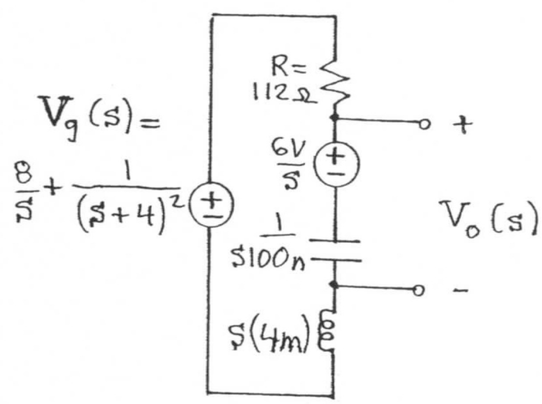
$t = 0^-$:



Because of the open circuit, $i_L(0^-) = 0$ A and there is no v-drop across R. Thus, the 6V is dropped across the open circuit, C.

$$v_C(0^-) = 6V$$

Using a v-source for the initial conditions on C, we have the following s-domain model:



Note: The $\frac{6V}{s}$ initial conditions for C must be included in $V_o(s)$.

c) We may use superposition to calculate $V_o(s)$.

With $V_g(s)$ on and $\frac{6V}{s}$ off (= wire) we have a V-divider.

$$V_{o1}(s) = V_g(s) \cdot \frac{\frac{1}{sC}}{sL + R + \frac{1}{sC}}$$

With $\frac{6V}{s}$ on and $V_g(s)$ off (= wire), we also have a V-divider. $V_o(s)$ equals the V-drop across R and sL.

$$V_{o2}(s) = \frac{6V}{s} \frac{sL + R}{sL + R + \frac{1}{sC}}$$

Combining results, $V_o(s) = V_{o1}(s) + V_{o2}(s)$.

$$V_o(s) = \frac{V_g(s) \cdot \frac{1}{sC} + \frac{6V}{s}(sL+R)}{sL + R + \frac{1}{sC}}$$

or

$$V_o(s) = \frac{\left[\frac{8}{s} + \frac{1}{(s+4)^2}\right]\frac{1}{s\,100n} + \frac{6V}{s}\left[s(4m) + 112\right]}{s(4m) + 112 + 1/(s\,100n)}$$

d) We use the initial value theorem:

$$\lim_{t \to \infty} v_o(t) = \lim_{s \to 0} sV_o(s)$$

The first step is to clear the $\frac{1}{s}$ in the denominator by multiplying $V_o(s)$ by s/s.

$$sV_o(s) = s\frac{\left(\left[\frac{8}{s} + \frac{1}{(s+4)^2}\right]\frac{1}{C} + 6V\left[sL+R\right]\right)}{s^2L + Rs + 1/C}$$

$$\lim_{s \to 0} sV_o(s) = \frac{\left[8 + \frac{s}{(s+4)^2}\right]\frac{1}{C} + 6V(s)(sL+R)}{s^2 L + Rs + 1/C}$$

Now we may set $s=0$ without having any divide-by-zero problems.

$$\lim_{s \to 0} sV_o(s) = \frac{\left[8 + \cancelto{0}{\frac{0}{16}}\right]\frac{1}{C} + \cancelto{0}{6V(0)R}}{0 + 0 + \frac{1}{C}}$$

$$'' \quad = 8 \, \frac{\frac{1}{C}}{\frac{1}{C}} = 8$$

Thus, $\lim_{t \to \infty} v_o(t) = 8 \text{ V}$