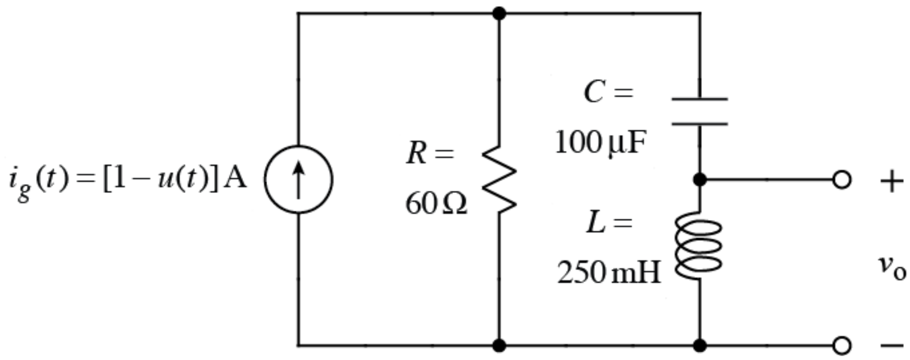
# HOMEWORK #14 probs 3 and 4 solution

Ex:




**Note:** The 1A in the $i_g(t)$ source is always on.

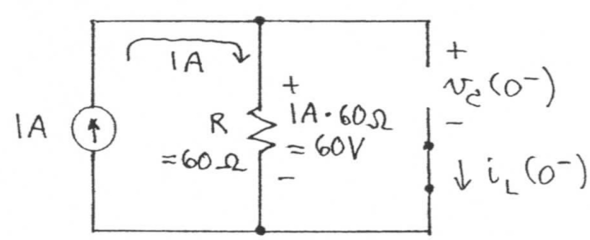
a) Write the Laplace transform $I_g(s)$ of $i_g(t)$.

b) Write the Laplace transform $V_o(s)$ of $v_o(t)$. Be sure to include the effects of initial conditions, if they are nonzero.

c) Write a numerical time-domain expression for $v_o(t)$ where $t \geq 0$.

SOL'N: a) $\mathcal{L}\{1 - u(t)\text{ A}\} = \frac{1}{s} - \frac{1}{s} = 0$

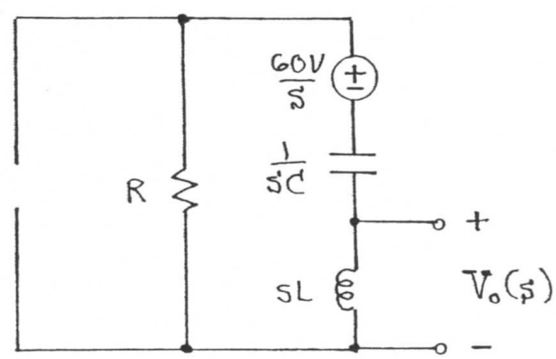
b) We find initial conditions at $t = 0^-$. We have a 1A DC input, and we treat L as a wire and C as an open circuit.




Because of the open-circuit C, $i_L(0^-) = 0$ A. All of the 1A flows thru R, creating 60V across R, which appears across C.

$$v_C(0^-) = 60\text{ V}$$

Using a series V-source, (we could use a parallel I-source instead), we have the following circuit model:




This is a V-divider.

$$V_o(s) = -\frac{60\,V}{s}\ \frac{sL}{sL + R + \frac{1}{sC}}$$

or

$$V_o(s) = -60V\ \frac{s\left(\frac{1}{4}\right)}{s^2\left(\frac{1}{4}\right) + 60s + \frac{1}{100\mu}}$$

or

$$V_o(s) = \frac{-60s}{s^2 + 240s + 40k}$$

or

$$V_o(s) = \frac{-60s}{(s + \underset{\overset{\shortparallel}{a}}{120})^2 + \underset{\overset{\shortparallel}{\omega}}{160}^2}$$

c) The denominator indicates we will have a decaying cosine and perhaps a decaying sine. Thus, we write the numerator of $V_o(s)$ as follows:

$$-60s = A(s + 120) + B(160)$$

To match the coefficient of $s$, we must have

$$A = -60.$$

With this value of $A$, we have an extra $-60(120)$ that we use $B$ to cancel out.

$$-60(120) + B(160) = 0$$

or

$$B = \frac{60(120)}{160} = 45$$

Thus, $V_o(s) = -60 \dfrac{s+120}{(s+120)^2 + 160^2} + 45 \dfrac{160}{(s+120)^2 + 160^2}$.

$$v_o(t) = \left[-60 e^{-120t} \cos(160t) + 45 e^{-120t} \sin(160t)\right] u(t) \text{ V}$$