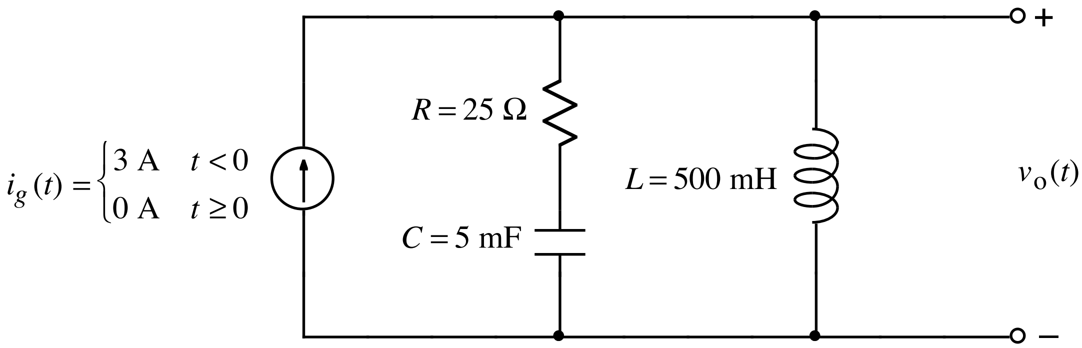
HOMEWORK #14 prob 5 solution

Ex:



a) Write the Laplace transform $I_g(s)$ of $i_g(t)$.

b) Write the Laplace transform $V_o(s)$ of $v_o(t)$. Be sure to include the effects of initial conditions, if they are nonzero.

c) Write a numerical time-domain expression for $v_o(t)$ where $t \geq 0$.

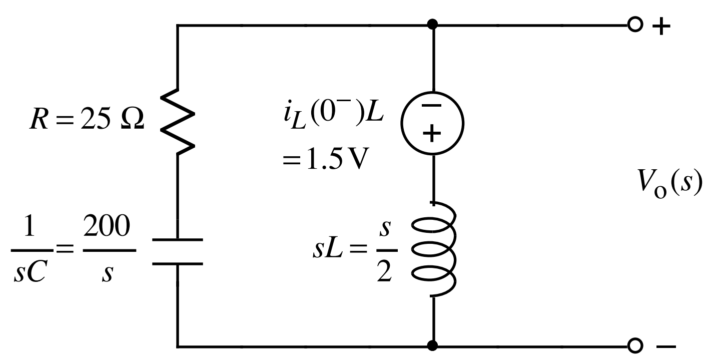
SOL'N: a) The Laplace transform depends only on what the input signal is from time 0 to ∞.

$$I_g(t) = \mathcal{L}\{i_g(t)\} = \mathcal{L}\{0\}\text{A} = 0\text{ A}$$

b) We find initial conditions by considering circuit at $t = 0^-$. The circuit has a 3 A input and has reached equilibrium. Thus, the L acts like a wire, shorting out the R and C. Thus, the initial conditions on the C are zero and all of the input current flows through the L.

$$i_L(0^-) = 3\text{ A}$$

Our circuit model includes initial conditions for the L but no input source, since the input source is zero:



The output voltage is found by using a voltage-divider formula. We observe that $V_o(s)$ is measured across the R and C. Thus, we may use a voltage-divider formula that avoids the need to add the voltage source to our answer. (An alternative approach is to use a voltage-divider to find the voltage across the L and then add the voltage source to our answer.)

$$V_o(s) = -1.5\,\mathrm{V}\frac{R + \dfrac{1}{sC}}{sL + R + \dfrac{1}{sC}} = -1.5\,\mathrm{V}\frac{25 + \dfrac{200}{s}}{\dfrac{s}{2} + 25 + \dfrac{200}{s}}$$

$$= -1.5\,\mathrm{V}\frac{50s + 400}{s^2 + 50s + 400}$$

$$= -75\,\mathrm{V}\frac{s + 8}{(s+10)(s+40)}$$

c) The output voltage versus time is the inverse Laplace transform of $V_o(s)$. We find a partial fraction expansion for the ratio of polynomials in s on the right side of the last expression above:

$$\frac{s+8}{(s+10)(s+40)} = \frac{A}{s+10} + \frac{B}{s+40}$$

Using the pole cover-up method, we compute A and B:

$$A = (s+10)\frac{s+8}{(s+10)(s+40)}\bigg|_{s=-10} = \frac{-10+8}{(-10+40)} = -\frac{1}{15}$$

$$B = (s+40)\frac{s+8}{(s+10)(s+40)}\bigg|_{s=-40} = \frac{-40+8}{(-40+10)} = \frac{16}{15}$$

Substituting into $V_o(s)$, we have the following partial fraction version:

$$V_o(s) = -5\,\mathrm{V}\left(\frac{-1}{s+10} + \frac{16}{s+40}\right)$$

Taking the inverse Laplace transform yields our final answer:

$$v_o(t \geq 0) = [5e^{-10t} - 80e^{-40t}]u(t)\ \mathrm{V}$$

NOTE: We could omit the $u(t)$, but it reminds us that our answer only applies to $t \geq 0$.