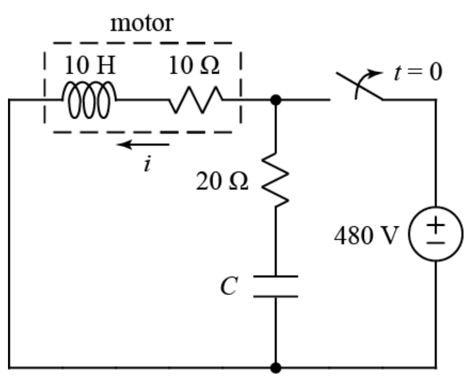
Ex:




After being closed for a long time, the switch opens at $t = 0$.

The above circuit represents a simplified circuit model of a motor and its power supply. The switch opens at time $t = 0$ to turn off the motor, and resistor $R_2$ and $C$ provide a discharge path for current in the motor.

a) Find the smallest **standard value** of $C$ that makes the circuit over-damped after the switch is opened at $t = 0$. For standard values, assume the value of $C$ is of form $\mathbf{1 \cdot 10^n}$ **F** or $\mathbf{2 \cdot 10^n}$ **F** or $\mathbf{5 \cdot 10^n}$ **F** where $n$ is an integer.

b) Using the $C$ value from (a), find a numerical expression for the motor current, $i(t)$, for $t > 0$.

sol'n: a) After $t = 0$, we have a series RLC circuit.

Characteristic roots are $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0^2 = \frac{1}{LC}$$

For over-damped roots we need $\alpha^2 > \omega_0^2$.

$$\alpha^2 = \left(\frac{R}{2L}\right)^2 = \left(\frac{30}{2(10)} \text{ r/s}\right)^2 = \left(\frac{3}{2}\right)^2 (\text{r/s})^2 = 2.25 \text{ r}^2/\text{s}^2$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{10C}, \quad \text{or} \quad C = \frac{1}{10\,\omega_0^2}$$

To achieve $\alpha^2 > \omega_0^2$ we need $C \geq \dfrac{1}{10(2.25)}$ F.

$$C \geq \frac{1}{10(2.25)} F = \frac{1}{22.5} F = 44.4 \ mF$$

The smallest $C \geq 44.4 \ mF$ with a standard value is $C = 50 \ mF$.

b) Using $C = 50 \ mF$, we re-calculate characteristic roots.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1.5 \pm \sqrt{2.25 - \frac{1}{10 \cdot 50m}}$$

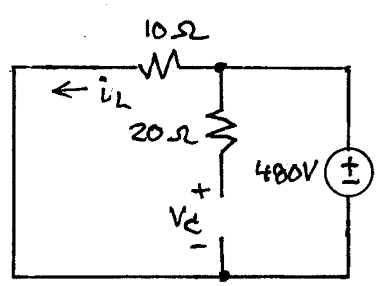
$$= -1.5 \pm \sqrt{2.25 - 2}$$

$$= -1.5 \pm \sqrt{0.25}$$

$$= -1.5 \pm 0.5$$

$$s_{1,2} = -1 \text{ and } -2$$

We find initial conditions at $t = 0^-$. Switch closed. L=wire, C=open




$$i_L = \frac{480V}{10\Omega} = 48A$$

$v_C = 480V$ since 20Ω R has no current

The form for the over-damped solution is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$$

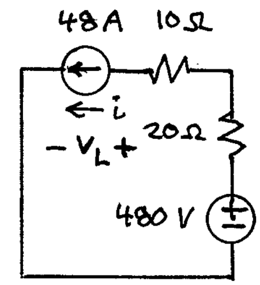
We will have $A_3 = 0 \ A$ since the circuit has no power supply on the left after $t = 0$.

Now we find $A_1$ and $A_2$.

$$i(0^+) = A_1 + A_2$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2 = -A_1 - 2A_2$$

Our circuit at $t=0^+$: $L = i_L$ source, $C = v_C$ source




$$i(0^+) = 48\text{ A}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L}$$

$$V_L = 480V - 48A(10\Omega + 20\Omega) = -960\text{ V}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{-960\text{ V}}{10\text{ H}} = -96\text{ A/s}$$

Matching the circuit values to the symbolic solution, we have the following:

$$A_1 + A_2 = 48\text{ A}$$
$$-A_1 - 2A_2 = -96\text{ A/s}$$

If we add the equations, we have

$$-A_2 = -48A \quad \text{or} \quad A_2 = 48\text{ A}$$

From the first eq'n, we see that we must have $A_1 = 0\text{ A}$

$$i(t>0) = 48e^{-2t}\text{ A}$$