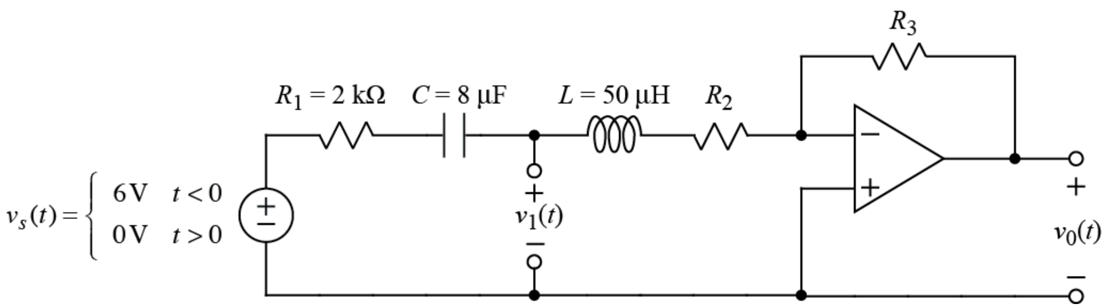
Ex:



The voltage source in the above circuit is off for $t > 0$.

a) Find a symbolic expression for the Laplace-transformed output, $\mathbf{V}_0(s)$, in terms of not more than R_1, R_2, R_3, L, C, and values of sources or constants.

b) Choose a numerical value for R_2 to make

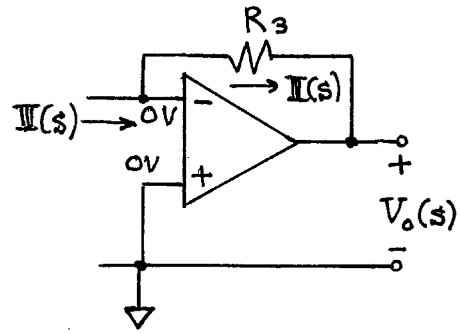
$$v_1(t) = v_m e^{-\alpha t} \cos(\beta t)$$

where v_m, α, and β are real-valued constants.

sol'n: a) No current flows into the op-amp inputs. Thus, current flowing toward the − input from the left will flow through R_3 (and into the op-amp then into the + or − power supply connections to the op-amp [which are not shown] then through the + or − power supply and back to the reference on the bottom rail [which was not shown]).

Since the op-amp has negative feedback, we expect that $V_- = V_+$ at the op-amp inputs. In other words, $V_- = V_+ = 0\text{V}$, and we have a virtual ground (or reference) at the − input of the op-amp.

We can find $V_o(s)$ from current, $I(s)$, flowing toward the − input of the op-amp.

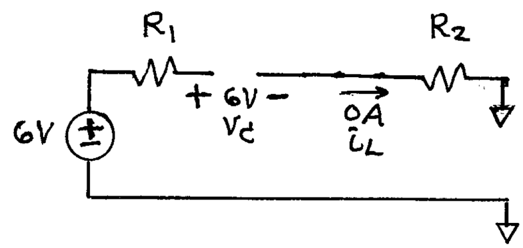


We have $V_o(s) = -I(s)\,R_3$.

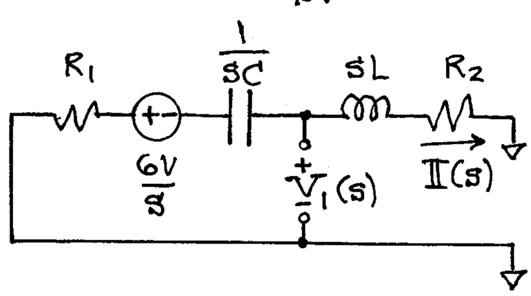
To find $I(s)$, we may treat the − input as reference.

First, however, we find initial conditions for the L and C.

$t=0^-$: $v_s(t) = 6V$, C = open, L = wire



We move to $t>0$ and include initial conditions on C. $V_s(t) = 0\,V$ = wire for $t>0$.



We have $I(s) = -\dfrac{6V}{s}\,\dfrac{1}{sL + R_1 + R_2 + \frac{1}{sC}}$

or $$\mathbb{I}(s) = \frac{-6V/L}{s^2 + \frac{R_1+R_2}{L}s + \frac{1}{LC}}$$

So $$V_o(s) = -\mathbb{I}(s) R_3$$

$$V_o(s) = \frac{6V\ R_3/L}{s^2 + \frac{R_1+R_2}{L}s + \frac{1}{LC}}$$

b) $V_1(s)$ is the same as the V-drop across L and R_2.

$$V_1(s) = \mathbb{I}(s)(sL + R_2)$$

or

$$V_1(s) = \frac{-(6V/L)(sL+R_2)}{s^2 + \frac{R_1+R_2}{L}s + \frac{1}{LC}}$$

or

$$V_1(s) = -6V\ \frac{s + R_2/L}{s^2 + \frac{R_1+R_2}{L}s + \frac{1}{LC}}$$

From the form of $v_1(t)$ given in the problem, we have another form for $V_1(s)$:

$$V_1(s) = \mathcal{L}\{v_m e^{-\alpha t}\cos(\beta t)\}$$

$$V_1(s) = v_m \frac{s+\alpha}{(s+\alpha)^2 + \beta^2}$$

$$V_1(s) = v_m \frac{s+\alpha}{s^2 + 2\alpha s + \alpha^2 + \beta^2}$$

Matching coefficients of the powers of s in the two forms of $V_1(s)$, we have the following equations:

$$R_2/L = \alpha, \quad \frac{R_1+R_2}{L} = 2\alpha, \quad \frac{1}{LC} = \alpha^2 + \beta^2$$

We have $\frac{R_2}{L} = \alpha = \frac{R_1 + R_2}{2L}$.

The solution is $R_2 = R_1 = 2k\Omega$.

$$R_2 = 2k\Omega$$