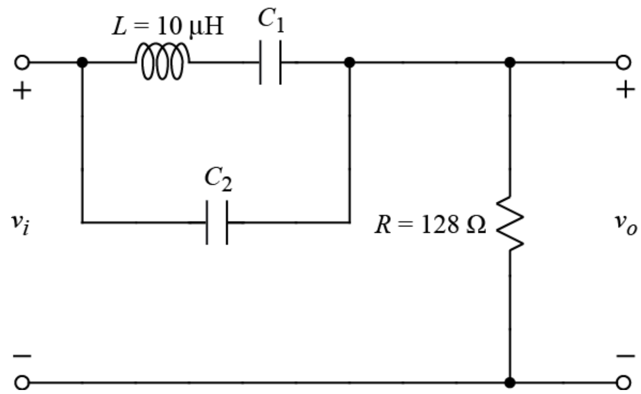
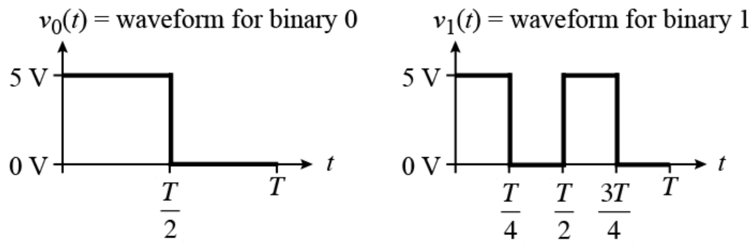
HOMEWORK #20 prob 4 solution

Ex:





T = one period of $v_0(t)$ = 6.28 μs

$$v_0(t) = \begin{cases} 5\text{ V} & 0 \le t < T/2 \\ 0\text{ V} & T/2 \le t < T \end{cases} \qquad v_1(t) = \begin{cases} 5\text{ V} & 0 \le t < T/4 \text{ and } T/2 \le t < 3T/4 \\ 0\text{ V} & T/4 \le t < T/2 \text{ and } 3T/4 \le t < T \end{cases}$$

The above filter circuit is being used in a communication system to detect whether received signals represent binary zeros or binary ones. The waveforms are designed to be detected using a method that determines whether the signal is high or low in each segment of length $T/4$, but only the analog filter circuit shown above is available.

The plan is to use the filter shown above to detect waveforms for zeros by designing the filter to pass the fundamental frequency, $\omega_0 = 2\pi/T$, of $v_0(t)$ but block the fundamental frequency, $2\omega_0$, of $v_1(t)$. (Presumably a different filter would be designed to detect waveforms for ones, but that filter is of no concern here.)

a) Find values of $C_1 \neq 0$ and $C_2 \neq 0$ such that the magnitude of the filter's transfer function, H, equals one for the fundamental frequency, ω_0, of $v_0(t)$ and zero for frequency $2\omega_0$.

b) Find the magnitude of the filter transfer function for frequency $6\omega_0$. (This frequency is present in $v_1(t)$, and it is desirable that it also be suppressed to a large degree.)

sol'n: a) We want $H(j\omega_0) = 1$ and $H(j2\omega_0) = 0$.

We achieve $H=1$ when L and C_1 are at resonance.

$$\omega_0^2 = \frac{1}{LC_1} \quad \text{or} \quad C_1 = \frac{1}{L\omega_0^2}$$

The fundamental frequency is $\omega_0 = \frac{2\pi}{T}$

or $\omega_0 = \frac{2\pi}{6.28\,\mu s} \approx 1\,M\,r/s$.

So $C_1 = \frac{1}{10\mu \cdot (1M)^2} F = \frac{1}{10}\mu F$ or $100\,nF$.

We achieve $H=0$ when $\frac{1}{j\omega C_2} \parallel \left(\frac{1}{j\omega C_1} + j\omega L\right) = \infty$.

This means we want the denominator of the parallel impedance (product over sum) to be zero.

Thus, $\frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} + j\omega L = 0$ @ $\omega = 2\omega_0$.

$$\frac{1}{j2\omega_0 C_2} = -\frac{1}{j2\omega_0 C_1} - j2\omega_0 L$$

$$\text{or} \quad j2\omega_0 C_2 = \frac{1}{-\frac{1}{j2\omega_0 C_1} - j2\omega_0 L}$$

$$C_2 = \frac{1}{-\frac{j2\omega_0}{j2\omega_0 C_1} - (j2\omega_0)^2 L}$$

$$C_2 = \frac{1}{-\frac{1}{C_1} + 4\omega_0^2 L}$$

$$C_2 = \frac{1}{-\frac{1}{100n} + 4(1M)^2 \cdot 10\mu} \, F$$

$$C_2 = \frac{1}{-10M + 40M} \, F$$

$$C_2 = \frac{1}{30M} \, F = \frac{1}{30} \, \mu F \text{ or } 33.3 \, nF$$

b) We calculate $H(j6\omega_o)$ as a voltage divider without the input voltage source:

$$H(j6\omega_o) = \frac{R}{\frac{1}{j6\omega_o C_2} \,\|\, \left(\frac{1}{j6\omega_o C_1} + j6\omega_o L\right) + R}$$

$$= \frac{128}{\frac{1}{j6M \cdot \frac{1}{30}\mu} \,\|\, \left(\frac{1}{j6M \cdot 100n} + j6M \cdot 10\mu\right) + 128}$$

$$= \frac{128}{-j5 \,\|\, \left(-j\frac{5}{3} + j60\right) + 128}$$

$$= \frac{128}{128 + -j\frac{15}{3} \,\|\, j\frac{175}{3}} = \frac{128}{128 + -j\frac{5}{3} \cdot 3 \,\|\, 35} = \frac{128}{128 - j\frac{165}{38}}$$

$$H(j6\omega_o) \doteq 1$$

The filter does not work very well for filtering out $6\omega_o$.