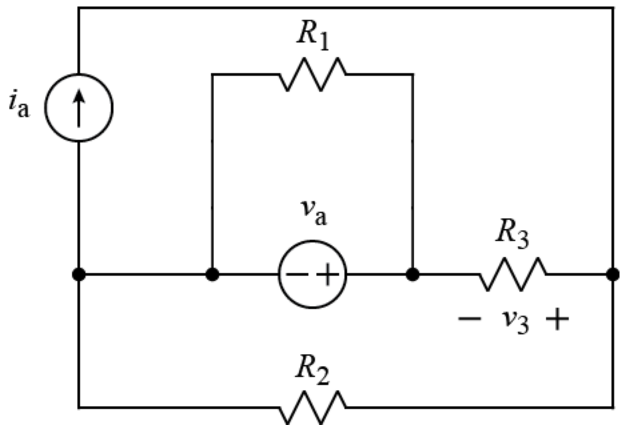
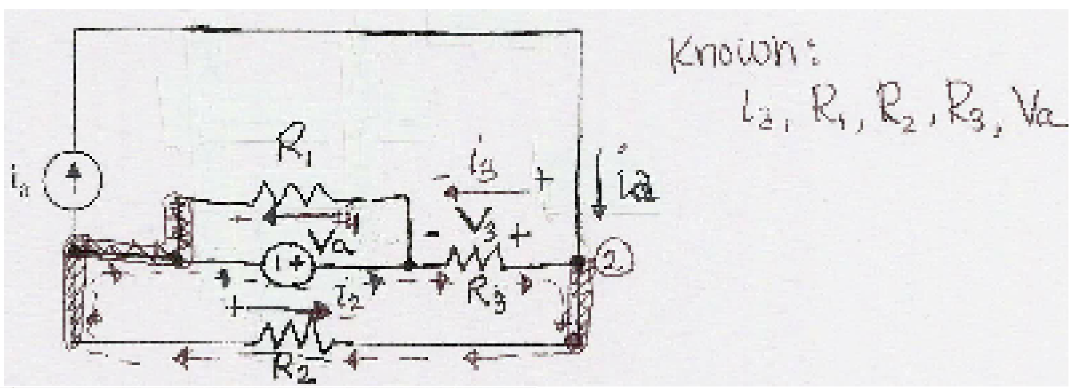
EX:

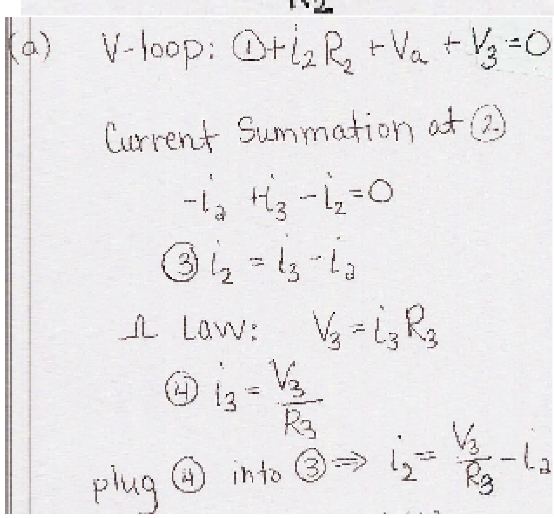


a) Derive an expression for v_3 containing not more than circuit parameters v_a, i_a, R_1, R_2, and R_3.

b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

SOL'N:




(a) V-loop: ① $+i_2 R_2 + V_a + V_3 = 0$

Current Summation at ②

$-i_a + i_3 - i_2 = 0$

③ $i_2 = i_3 - i_a$

Ω Law: $V_3 = i_3 R_3$

④ $i_3 = \frac{V_3}{R_3}$

plug ④ into ③ $\Rightarrow$ $i_2 = \frac{V_3}{R_3} - i_a$

plug into ① $\Rightarrow$ $\dfrac{V_3(R_2)}{R_3} - i_a R_2 + V_a + V_3 = 0$

$$\boxed{V_3 = \frac{i_a R_2 - V_a}{1 + \frac{R_2}{R_3}}} \quad \text{OR} \quad \frac{(i_a R_2 - V_a) R_3}{R_3 + R_2}$$

b) A consistency check is accomplished by making certain component values zero, in order to simplify the circuit enough that it may be solved by inspection. The zero values are then substituted into the solution given by the complete formula from (a) to verify that it yields the result found by inspection. If enough such checks are performed and are passed, then the solution in part (a) is probably correct.

There are many possible checks. For example, if we set R_3 to zero, it becomes a wire with no voltage drop. Thus, the answer for v_3 must be zero. For the complete solution, we would get the following calculation:

$$v_3 = \frac{(i_a R_2 - v_a) R_3}{R_2 + R_3} = \frac{(i_a R_2 - v_a) \bullet 0}{R_2 + R_3} = 0 \quad \text{solution verified}$$

Another possible check is $v_a = 0$, which turns the v_a source into a wire that bypasses R_1 and reduces the circuit to a current divider involving only i_a, R_2, and R_3. We can write down a formula for the current in R_3 and then use Ohm's law to find v_3:

$$i_3 = \frac{i_a R_2}{R_2 + R_3}$$

$$v_3 = i_3 R_3 = \frac{i_a R_2}{R_2 + R_3} R_3$$

If we plug $v_a = 0$ into the solution from (a) we get the same result, and the solution from is verified as satisfying this special case.

Yet another possible check is $i_a = 0$, which turns the i_a source into an open circuit, leaving v_a across R_2, and R_3 and forming a voltage divider. (Notice that R_1 is a second circuit across the v_a source that may be solved separately.) Using a voltage divider formula, we have the following value for v_3:

$$v_3 = \frac{v_a R_3}{R_2 + R_3}$$

If we plug $i_a = 0$ into the solution from (a) we get the same result, and the solution from is again verified as satisfying this special case.