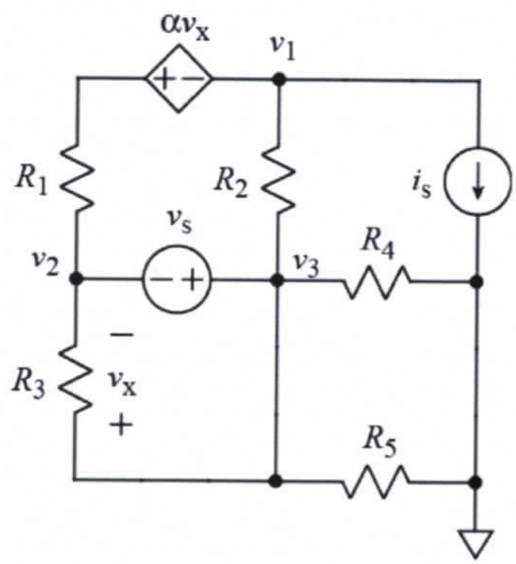
EXAM #1 prob 4 solution

Ex:



For the circuit shown, write (but do not solve) three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

sol'n: We use the node-v method. We observe that v_2 and v_3 are connected by only a v-source and so form a super-node. For the dependent source, we define v_x in terms of node voltages. This is a good place to start.

$$v_x = v_3 - v_2 = v_s$$ (This last step is optional but helpful.)

Now we can write an eq'n for node v_1.

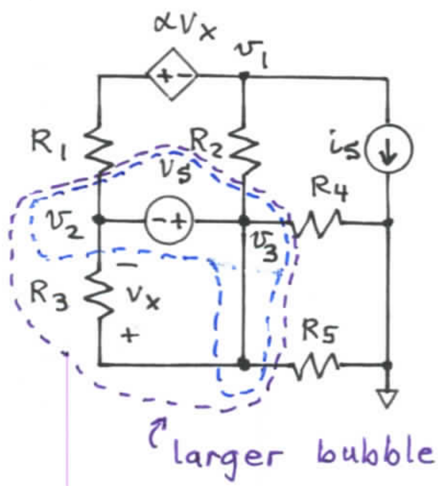
(1) $$\frac{v_1 + \alpha\overbrace{(v_3 - v_2)}^{v_x} - v_2}{R_1} + \frac{v_1 - v_3}{R_2} + i_s = 0\,A$$

Note that we could also replace the $v_3 - v_2$ term with v_s.

The super-node for v_2 and v_3 has a voltage eq'n and a current eq'n. The voltage eq'n is simple.

(2) $v_s = v_3 - v_2$ or $v_2 + v_s = v_3$

For the super-node current summation, we start with a bubble around v_2, v_3, and v_s. Closer inspection, however, reveals that R_3 also connects v_2 to v_3. Thus, the current in R_3 will be computed in both directions and will cancel out in the i-sum. So we put R_3 in the bubble, too.



The sum out of the bubble is as follows:

(3) $$\frac{v_2 - [v_1 + \alpha(v_3 - v_2)]}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3}{R_4} + \frac{v_3}{R_5} = 0\,A$$

Eq'ns (1) - (3) are the desired answer.