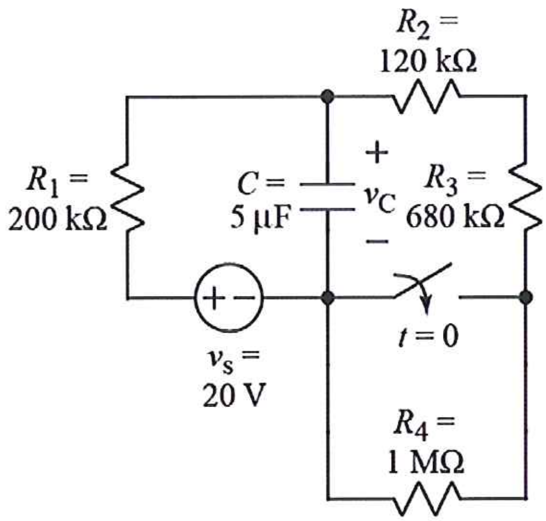
EXAM #2 prob 2 solution

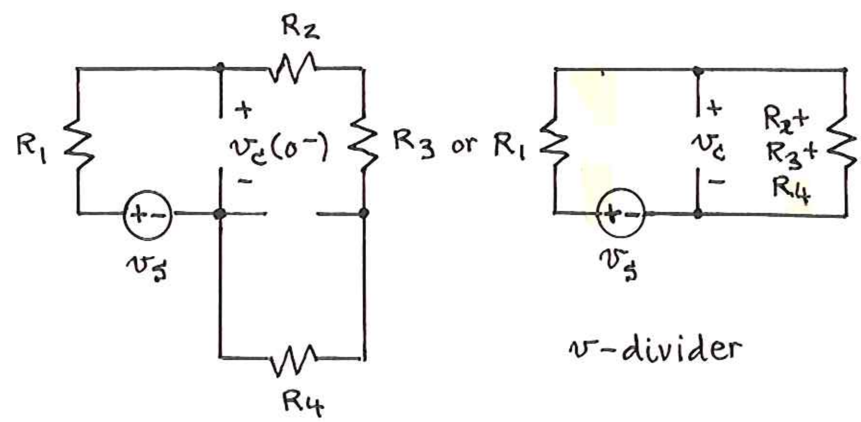
Ex:



After being open for a long time, the switch closes at $t = 0$.

Write a numerical time-domain expression for $v_C(t > 0)$, the voltage across C.

soln: $t = 0^-$: C = open, switch open, find $v_C(0^-)$



Using v-divider: $v_C(0^-) = v_s \dfrac{R_2 + R_3 + R_4}{R_1 + R_2 + R_3 + R_4}$

or

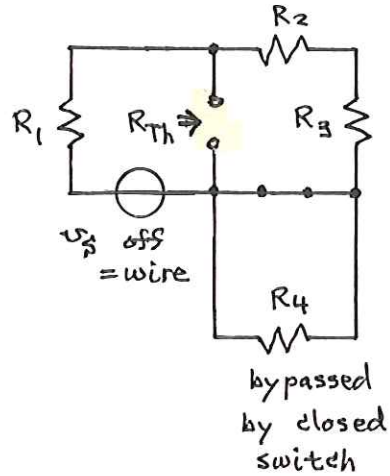
$$v_C(0^-) = 20V \cdot \frac{1.8\ M\Omega}{2\ M\Omega}$$

or

$$v_C(0^-) = 18\ V$$

$t=0^+$: $v_c(0^+) = v_c(0^-) = 18\,V$ No circuit work this time.

$t>0$: Find R_{Th} for circuit where C is connected. Turn off independent source v_s. Look into circuit from where C is connected.



R_1 is in parallel with $R_2 + R_3$.

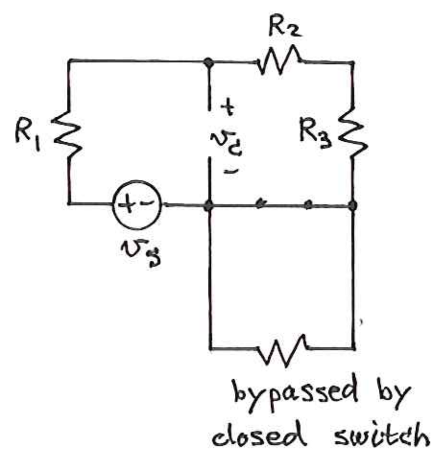
$$R_{Th} = R_1 \| (R_2 + R_3) = 200\,k\Omega \| (120\,k\Omega + 680\,k\Omega)$$

or

$$R_{Th} = 200\,k\Omega \| 800\,k\Omega = 200\,k\Omega \cdot 1\|4 = 160\,k\Omega$$

So $\tau = R_{Th}C = 160\,k\Omega \cdot 5\,\mu F = 800\,ms$.

$t\to\infty$: C = open, switch closed, find $v_c(t\to\infty)$



v_c is across R_2+R_3 in a v-divider

$$v_c(t\to\infty) = v_s \frac{R_2+R_3}{R_1+R_2+R_3}$$

$$= 20V \cdot \frac{120k + 680\,k\Omega}{200K + 120K + 680\,K\Omega}$$

$$= 20V \cdot \frac{800k}{1\,M\Omega} = 16V$$

Summary of results:

$$v_c(0^+) = 18V$$

$$\tau = 800\,ms$$

$$v_c(t \to \infty) = 16V$$

Now we use the general soln for RC circuits:

$$v_c(t>0) = v_c(t \to \infty) + \left[v_c(0^+) - v_c(t \to \infty)\right] e^{-t/\tau}$$

or

$$v_c(t>0) = 16V + (18V - 16V)\, e^{-t/800\,ms}$$

or

$$v_c(t>0) = 16V + 2V\, e^{-t/800\,ms}$$