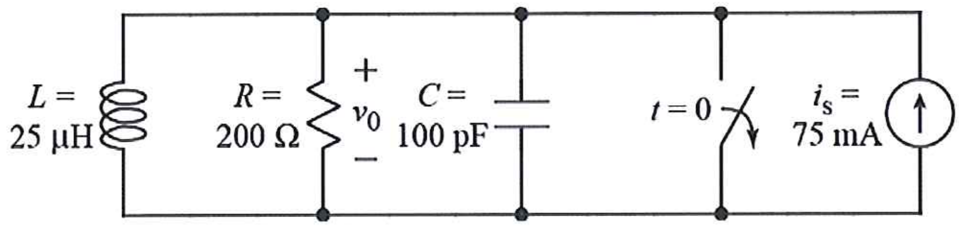
ECE 2240
F 13

EXAM #2 prob 3 solution

Ex:



After being closed for a long time, the switch opens at $t = 0$.

The inductor carries no current at $t = 0^-$.

The characteristic roots of the circuit are

$$s_1 = -20\,\text{Mr/s} \quad \text{and} \quad s_2 = -30\,\text{Mr/s}.$$

The voltage across the resistor, v_0, has the following over-damped form:

$$v_0(t > 0) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3 \quad \text{where } A_3 = 0\text{ V}$$

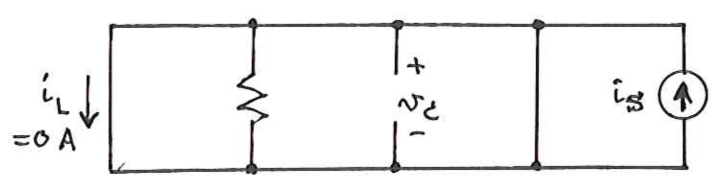
a) Find the numerical value of $v_0(t = 0^+)$.

b) Find the numerical value of $\left.\dfrac{dv_0(t)}{dt}\right|_{t=0^+}$

c) Using the answers for (a) and (b), find the numerical values of A_1 and A_2. Note: if you do not have values for (a) and (b), make up non-zero values for them and solve the problem (for partial credit).

sol'n: a) To determine what is happening at $t=0^+$, we find the energy variables $i_L(0^-)$ and $v_C(0^-)$ at $t=0^-$. The value of $i_L(0^-)$ is given as zero in the problem statement. (Without this information we would be unable to determine how i_s splits between the L and the switch.)

$t=0^-$: L = wire, C = open, switch closed



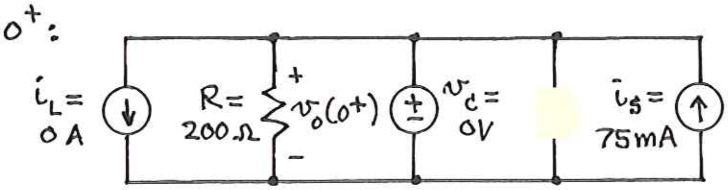
The C is shorted by both the L and the switch. Thus, $v_C(0^-) = 0V$.

Since i_L and v_C are energy variables, they will not change instantly.

$$i_L(0^+) = i_L(0^-) = 0A \text{ and } v_C(0^+) = v_C(0^-) = 0V$$

At $t=0^+$ we treat the L and C as sources.

$t=0^+$:



In this circuit, the R is in parallel with v_C and has the same v-drop as C.

$$v_o(0^+) = v_C(0^+) = 0V$$

b) To find the value of $\left.\frac{dv_o(t)}{dt}\right|_{t=0^+}$, we express v_o as a function of energy variables i_L and v_C. Then we differentiate i_L and v_C on one side of the expression and use $\frac{di_L}{dt} = \frac{v_L}{L}$ and/or $\frac{dv_C}{dt} = \frac{i_C}{C}$ to translate the derivatives into non-derivatives. On the other side of the expression for v_o we differentiate to get dv_o/dt. At this point, we have an expression for $\left.\frac{dv_o}{dt}\right|_{t=0^+}$ in terms of $v_L(0^+)$ and/or $i_C(0^+)$, which we can find from our model of $t=0^+$.

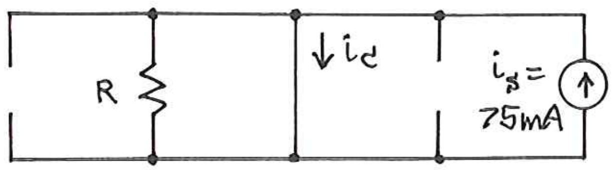
Here, $v_o(t) = v_C(t)$ since they are in parallel.

So $\frac{dv_o(t)}{dt} = \frac{dv_C(t)}{dt} = \frac{i_C(t)}{C}$.

Evaluate at $t=0^+$: $\left.\frac{dv_o(t)}{dt}\right|_{t=0^+} = \frac{i_C(t=0^+)}{C}$

Going back to our $t=0^+$ model, we see that the L is an open and C is a short.

$t=0^+$:



The current from i_s will all flow thru C.

$$i_C(0^+) = i_s = 75 \text{ mA}$$

So

$$\left.\frac{dv_o(t)}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{75 \text{ mA}}{100 \text{ pF}} = 750 \text{ MV/s}$$

c) For the symbolic sol'n we have

$$v_o(0^+) = A_1 + A_2 \text{ and } \left.\frac{dv_o(t)}{dt}\right|_{t=0^+} = A_1 s_1 + A_2 s_2$$

Thus,

$$A_1 + A_2 = 0\text{V} \quad \text{and} \quad A_1(-20\text{Mr/s}) + A_2(-30\text{Mr/s}) = 750 \text{ MV/s}$$

or

$$A_2 = -A_1$$

so

$$A_1(-20) - A_1(-30) = 750\text{V}$$

or

$$A_1(30-20) = 750\text{V}$$

or

$$A_1 = \frac{750\text{V}}{10} = 75\text{V}$$

and

$$A_2 = -A_1 = -75\text{V}$$