EXAM #3 prob 1 solution

Ex: Find $V(s) = \mathcal{L}\left\{\frac{d}{dt}[4t\sin(8t)]\right\}$ where $\mathcal{L}$ means "Laplace transform."

SOL'N: We could start halfway down the page, but we peel away the layers of the onion to work our way down to the starting point. First, we consider the identity for differentiation:

$$\mathcal{L}\left\{\frac{d}{dt}[4t\sin(8t)]\right\} = s\mathcal{L}\{4t\sin(8t)\} - 4t\sin(8t)\big|_{t=0^-}$$

or

$$\mathcal{L}\left\{\frac{d}{dt}[4t\sin(8t)]\right\} = s\mathcal{L}\{4t\sin(8t)\} - 0$$

Now we observe that we can apply the identity for multiplication by t:

$$\mathcal{L}\{4t\sin(8t)\} = -\frac{d}{ds}\mathcal{L}\{4\sin(8t)\}$$

We are now at the innermost term, which we can look up in a table of transform pairs.

$$\mathcal{L}\{4\sin(8t)\} = 4\cdot\frac{8}{s^2+8^2}$$

Now we work our way back out. We apply the identity for multiplication by t, and we use some calculus tricks to avoid the quotient rule for d/dt.

$$\mathcal{L}\{4t\sin(8t)\} = -\frac{d}{ds}\left(4\cdot\frac{8}{s^2+8^2}\right) = -32\frac{d}{ds}\left[s^2+8^2\right]^{-1}$$

We use the rule for differentiation of polynomials

$$\mathcal{L}\{4t\sin(8t)\} = -32(-1)\left[s^2+8^2\right]^{-2}(2s) = \frac{64s}{\left(s^2+8^2\right)^2}$$

Taking the derivative means we multiply by s, (and initial conditions are zero as shown at the beginning of the solution).

$$\mathcal{L}\left\{\frac{d}{dt}[4t\sin(8t)]\right\} = \frac{64s^2}{\left(s^2+8^2\right)^2}$$