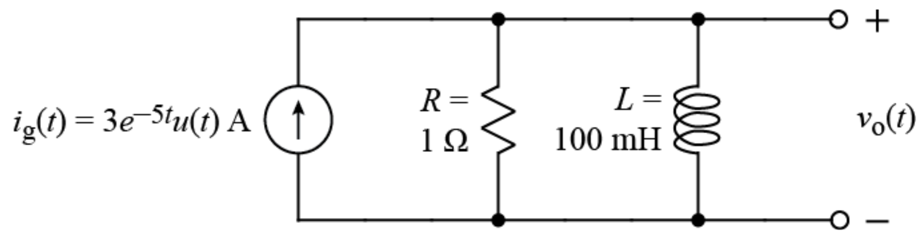
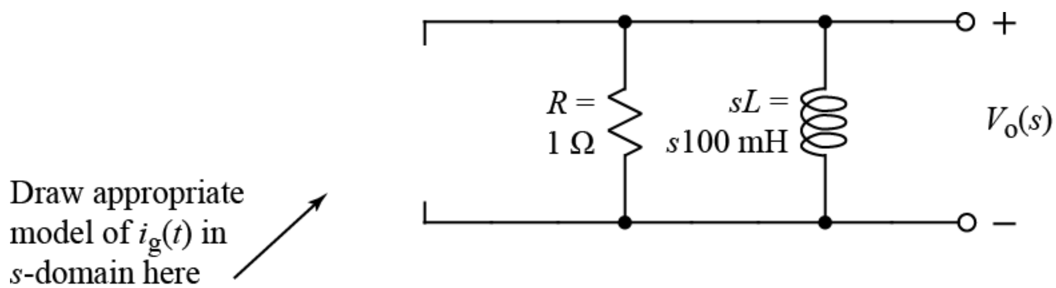
Ex:



a) For the circuit shown above, find the Laplace transform $I_g(s)$ of $i_g(t)$ and draw an appropriate representation of the current source on the s-domain circuit diagram below.



b) The Laplace transform of the output signal of the circuit is the following:

$$V_o(s) = \frac{3s}{s^2 + 15s + 50}$$

Write a numerical time-domain expression for $v_o(t)$ where $t \geq 0$.

SOL'N: a) The scaling factor of 3 in the time domain scales the answer in the s-domain, and the $u(t)$ has no effect on the Laplace transform. Thus, we merely scale the Laplace transform of e^{-5t} (found in a table of transform pairs) by a factor of 3.

$$I_g(s) = \mathcal{L}\left\{i_g(t)\right\} = \mathcal{L}\left\{3e^{-5t}u(t)\,\mathrm{A}\right\} = \frac{3}{s+5}\,\mathrm{A}$$

In the circuit, the source symbol is the same but is labeled with the value of the Laplace transform. Units may be omitted.

b) The denominator has real roots, and we have a straightforward partial fraction problem.

$$V_o(s) = \frac{3s}{s^2 + 15s + 50} = \frac{A}{s+5} + \frac{B}{s+10}$$

The pole-coverup method yields the values of A and B.

$$A = (s+5)V_o(s)\Big|_{s=-5} = (s+5)\frac{3s}{(s+5)(s+10)}\Bigg|_{s=-5}$$

or

$$A = \frac{3s}{(s+10)}\Bigg|_{s=-5} = \frac{3(-5)}{(-5+10)} = \frac{-15}{5} = -3$$

and

$$B = (s+10)V_o(s)\Big|_{s=-10} = (s+10)\frac{3s}{(s+5)(s+10)}\Bigg|_{s=-10}$$

or

$$B = \frac{3s}{(s+5)}\Bigg|_{s=-10} = \frac{3(-10)}{(-10+5)} = \frac{-30}{-5} = 6$$

Summary:

$$V_o(s) = \frac{-3}{s+5} + \frac{6}{s+10}$$

We take the inverse transform of each term, multiply by $u(t)$ to remind ourselves that we don't know what is happening before $t = 0$, and add units.

$$v_o(t) = \left[-3e^{-5t} + 6e^{-10t}\right]u(t)\,\mathrm{V}$$