HOMEWORK #11 prob 2 solution

Ex: Compute the Laplace transform of the following functions by calculating the integral expression for the Laplace transform (step-by-step by hand):

a) $f(t) = u(t) - u(t-1)$ where $u(t)$ is the unit step function: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t \end{cases}$

b) $f(t) = \begin{cases} 0 & t < 0 \\ e^{-2t} & 0 \le t \end{cases}$

c) $f(t) = \begin{cases} 0 & t < 4 \\ e^{-2(t-4)} & 4 \le t \end{cases}$

SOL'N: a) We can separate the integral into two pieces.

$$F(s) = \int_{0^-}^{\infty} [u(t) - u(t-1)]e^{-st}\,dt = \int_{0^-}^{\infty} u(t)e^{-st}\,dt - \int_{0^-}^{\infty} u(t-1)e^{-st}\,dt$$

The step function is equal to unity or zero and has the effect of changing the limits on the integral.

$$F(s) = \int_{0^-}^{\infty} e^{-st}\,dt - \int_{1}^{\infty} e^{-st}\,dt = \left.\frac{e^{-st}}{-s}\right|_{0^-}^{\infty} - \left.\frac{e^{-st}}{-s}\right|_{1}^{\infty}$$

We assume that the value of s has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = \frac{0}{-s} - \frac{e^{-s0^-}}{-s} - \left[\frac{0}{-s} - \frac{e^{-s\cdot 1}}{-s}\right] = \frac{1}{s} - \frac{e^{-1}}{s} = \frac{1 - e^{-1}}{s}$$

b)

$$F(s) = \int_{0^-}^{\infty} e^{-2t}e^{-st}\,dt = \left.\frac{e^{-(2+s)t}}{-(2+s)}\right|_{0^-}^{\infty} = \frac{0}{-(2+s)} - \frac{e^{0}}{-(2+s)}$$

We assume that the value of s has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = \frac{1}{s+2}$$

c) We set the lower limit of the integral to zero to account for the part of $f(t)$ that is zero.

$$F(s) = \int_4^{\infty} e^{-2(t-4)} e^{-st}\, dt = \int_4^{\infty} e^{-2t} e^{8} e^{-st}\, dt = e^{8} \int_4^{\infty} e^{-2t} e^{-st}\, dt$$

As before, we assume that the value of s has a positive real part so that the upper limit of the integral equals zero.

$$F(s) = e^{8} \left. \frac{e^{-(2+s)t}}{-(2+s)} \right|_4^{\infty} = e^{8} \left[\frac{0}{-(2+s)} - \frac{e^{-(2+s)4}}{-(2+s)} \right] = \frac{e^{-4s}}{s+2}$$