



EX: Using algebraic simplification and a table of Laplace transform pairs, find the inverse Laplace transform of each of the following functions:

a) $F(s) = 3e^{-(s-1)}$

b) $F(s) = \frac{1}{s} + \frac{3}{s^2 + 4}$

c) $F(s) = \frac{s+5}{s^2 + 2s + 5}$

SOL'N: a)

$$\mathcal{L}^{-1}\{3e^{-(s-1)}\} = 3e\mathcal{L}^{-1}\{e^{-s}\} = 3e\delta(t-1)$$

SOL'N: b)

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{3}{s^2 + 4}\right\} &= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2 + 2^2}\right\} \\ &= u(t) + \frac{3}{2}\sin(2t)u(t)\end{aligned}$$

SOL'N: c)

$$\begin{aligned}\mathcal{L}^{-1}\left\{\frac{s+5}{s^2 + 2s + 5}\right\} &= \mathcal{L}^{-1}\left\{\frac{s+1+2(2)}{(s+1)^2 + 2^2}\right\} \\ &= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2 + 2^2}\right\} + 2\mathcal{L}^{-1}\left\{\frac{2}{(s+1)^2 + 2^2}\right\} \\ &= [e^{-t}\cos(2t) + 2e^{-t}\sin(2t)]u(t)\end{aligned}$$