HOMEWORK #12 prob 1b solution

EX: Find the Laplace transform of

$$\frac{d}{dt}\left[e^{-at}\sin(\omega t)u(t)\right]$$

SOL'N: Use the identity for derivatives:

$$\mathcal{L}\left\{\frac{d}{dt}[f(t)u(t)]\right\} = sF(s) - f(0^-)$$

where

$$F(s) \equiv \mathcal{L}\{f(t)\} \equiv \int_{0^-}^{\infty} f(t)e^{-st}dt$$

From a lookup table of transform pairs, we have the following:

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{e^{-at}\sin(\omega t)\right\} = \frac{\omega}{(s+a)^2+\omega^2}$$

For the initial value, we have zero:

$$f(0^-) = e^{-a\cdot 0^-}\sin(\omega\cdot 0^-) = 1\cdot 0 = 0$$

Our final result:

$$\mathcal{L}\left\{\frac{d}{dt}\left[e^{-at}\sin(\omega t)u(t)\right]\right\} = \frac{s\omega}{(s+a)^2+\omega^2}$$