EX: Find the Laplace transform of

$$\int_0^t te^{-at}\,dt$$

SOL'N: We use the integral identity:

$$\mathcal{L}\left\{\int_0^t f(t)dt\right\} = \frac{F(s)}{s}$$

From a table, we have $F(s)$

$$F(s) = \mathcal{L}\left\{te^{-at}\right\} = \frac{1}{(s+a)^2}$$

To obtain our final answer, we need only divide by s:

$$\mathcal{L}\left\{\int_0^t te^{-at}\,dt\right\} = \frac{1}{s(s+a)^2}$$

NOTE: We could compute the integral first and then Laplace transform, but this would be more tedious.