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Ex: Find $L\{tu(t-3)e^{-4t}\}$.

Sol'n: We use the time shift identity:

$$L\{f(t-a)u(t-a)\} = e^{-as}F(s)$$

The time-shifted step function, $u(t-3)$, dictates that the time delay is $a = 3$. To use the identity, we find a way to write the entire expression being transformed into a function of $t-3$. We achieve this by subtracting and adding 3 from and to t:

$$L\{tu(t-3)e^{-4t}\} = L\{([t-3]+3)u(t-3)e^{-4([t-3]+3)}\}$$

By replacing $t-3$ with t throughout, we find $f(t)$:

$$f(t) = (t+3)e^{-4(t+3)}$$

Now we Laplace transform $f(t)$:

$$F(s) = L\{te^{-4(t+3)} + 3e^{-4(t+3)}\}$$

or

$$F(s) = L\{te^{-12}e^{-4t} + 3e^{-12}e^{-4t}\}$$

or

$$F(s) = L\{te^{-12}e^{-4t} + 3e^{-12}e^{-4t}\}$$

or

$$F(s) = e^{-12}L\{te^{-4t}\} + 3e^{-12}L\{e^{-4t}\}$$

or

$$F(s) = e^{-12}\frac{1}{(s+4)^2} + 3e^{-12}\frac{1}{s+4}$$

Applying the identity yields the final answer:

$$L\{tu(t-3)e^{-4t}\} = e^{-3s}\left[e^{-12}\frac{1}{(s+4)^2} + 3e^{-12}\frac{1}{s+4}\right]$$