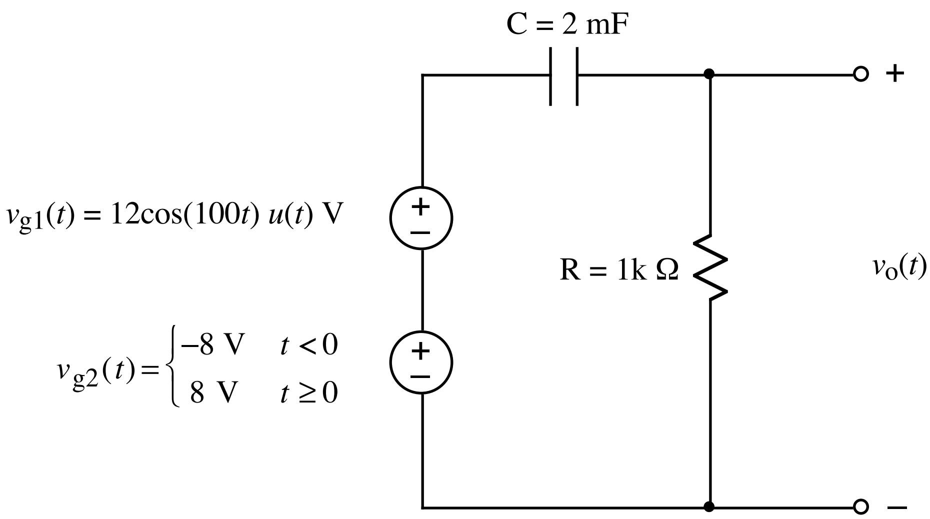
Ex:



a) Write the Laplace transform, $V_{g2}(s)$, of $v_{g2}(t)$.

b) Draw the s-domain equivalent circuit, including sources $V_{g1}(s)$ and $V_{g2}(s)$, components, initial conditions for C, and terminals for $V_o(s)$.

c) For the circuit in problem 3, write an expression for $V_o(s)$.

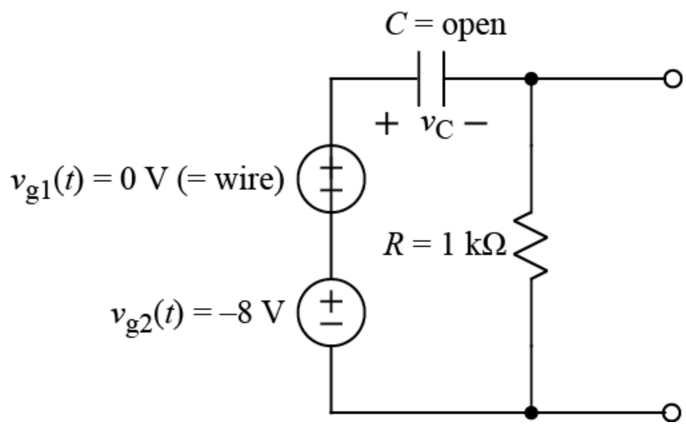
d) Apply the final value theorem to $V_o(s)$ to find $\lim_{t \to \infty} v_o(t)$.

SOL'N: a) The Laplace transfrom integral excludes events that happen before time zero, so we consider only what happens for $t \geq 0$. We treat the constant 8V after time zero as $8u(t)$:

$$V_{g2}(s) = \frac{8}{s}\text{V}$$

NOTE: Units become problematic when working with the Laplace transform because of the possible confusion between seconds, s, and the Laplace variable, *s*. When typing, italics font distinguishes the Laplace variable from the units of seconds, but when writing by hand, the two may difficult to distinguish. Consequently, units may sometimes be omitted in solutions written by hand.

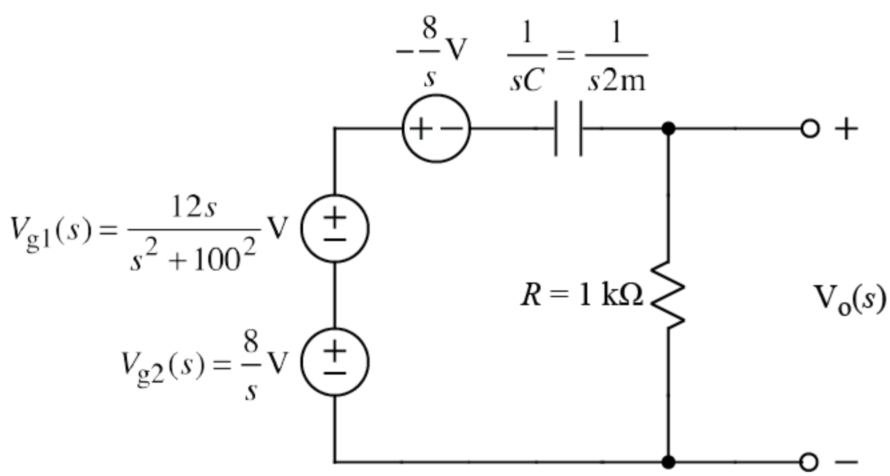
b) We find initial conditions by considering the circuit at $t = 0^-$. At that time, the v_{g1} source is zero, the v_{g2} source is a constant –8V, and the C acts like an open circuit.



We find only the energy variable $v_C(0^-)$. Since no current flows, the entire voltage drop of –8V will be across the capacitor:

$$v_C(t=0^-)=8\,\mathrm{V}$$

For convenience, we use a series voltage source for the initial conditions on *C*. Note that the polarity of the initial condition source is the same as that of the measured value at t = 0^-. Also, since voltage is the energy variable for *C*, the source acts as a step function that turn on at t = 0.



c) We have a voltage-divider driven by the sum of the voltage sources.

$$\mathrm{V_o}(s)=\left(\frac{12s}{s^2+100^2}\mathrm{V}+\frac{8}{s}\mathrm{V}--\frac{8}{s}\mathrm{V}\right)\frac{R}{R+\dfrac{1}{sC}}$$

or

$$\mathrm{V_o}(s) = \left(\frac{12s}{s^2 + 100^2}\mathrm{V} + \frac{16}{s}\mathrm{V} \right) \frac{1\mathrm{k}}{\frac{1}{s2\mathrm{m}} + 1\mathrm{k}}$$

d) We use the final value theorem:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s) = \lim_{s\to 0} s\left(\frac{12s}{s^2 + 100^2}\mathrm{V} + \frac{16}{s}\mathrm{V} \right) \frac{1\mathrm{k}}{\frac{1}{s2\mathrm{m}} + 1\mathrm{k}}$$

We multiply through by s in the term in parentheses:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} \left(\frac{12s^2}{s^2 + 100^2}\mathrm{V} + 16\,\mathrm{V} \right) \frac{1\mathrm{k}}{\frac{1}{s2\mathrm{m}} + 1\mathrm{k}}$$

We need a polynomial over a polynomial so we can identify the constant terms (which will be the only ones left when s goes to zero) on the top and bottom. To obtain the desired form, we multiply the term on the right by s/s:

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} \left(\frac{12s^2}{s^2 + 100^2}\mathrm{V} + 16\,\mathrm{V} \right) \frac{s1\mathrm{k}}{\frac{1}{2\mathrm{m}} + s1\mathrm{k}}$$

We may now plug in $s = 0$ since we have no indeterminant zero divided by zero terms that will result.

$$\lim_{t\to\infty} v_o(t) = \left(\frac{12(0)}{0 + 100^2}\mathrm{V} + 16\,\mathrm{V} \right) \frac{(0)1\mathrm{k}}{\frac{1}{2\mathrm{m}} + (0)1\mathrm{k}} = (16\,\mathrm{V})(0)$$

Our answer turns out to be zero, meaning the R has no voltage across it as time approaches infinity. This makes sense since the C will become an open circuit and no current will flow.

$$\lim_{t\to\infty} v_o(t) = 0\,\mathrm{V}$$

NOTE: We get an nozero answer for the final value only when we have a pole at zero in $\mathrm{V_o}(s)$. That is, we must have a $1/s$ term. In the present case, we appear to have a $1/s$ termin the $16/s$, but if we multiply the 16/*s* by the last term, the apparent pole at zero proves to be an illusion:

$$\frac{16}{s}\text{V}\frac{1\text{k}}{\dfrac{1}{s2\text{m}}+1\text{k}}=\frac{16\,\text{k}}{\dfrac{1}{2\text{m}}+s1\text{k}}$$