HOMEWORK #15 prob 1 solution

Ex: Give numerical answers to each of the following questions:

a) Find the value of $z = 12 - j16 + 7 + j24$.

b) Find the magnitude of $z = 15 + j8$.

c) Find the conjugate of $z = \dfrac{3+j4}{j} \cdot \dfrac{-j}{3-j4}$.

d) Find the value of $z = \left(1 + j\sqrt{3}\right)\left(\dfrac{\sqrt{3}}{4} - j\dfrac{1}{4}\right)$.

Sol'n: a) Sum the real parts, and sum the imaginary parts.

$$z = 12 + 7 - j16 + j24 = 19 + j8$$

b) Think of the complex number as a vector. Use the Pythagorean theorem to find the magnitude (or length) of this vector.

$$|z| = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

c) We use an asterisk to designate a conjugate. To find the conjugate, we change each *j* to –*j*.

$$z^* = \left(\frac{3+j4}{j} \cdot \frac{-j}{3-j4}\right)^* = \frac{3-j4}{-j} \cdot \frac{j}{3+j4} = \frac{j}{-j} \cdot \frac{3-j4}{3+j4} = -1 \cdot \frac{3-j4}{3+j4}$$

Now we rationalize the denominator by multiplying top and bottom by the conjugate of the denominator.

$$z^* = -\frac{3-j4}{3+j4} \cdot \frac{3-j4}{3-j4} = -\frac{9-16-j12-j12}{3^2+4^2} = \frac{7+j24}{25} = \frac{7}{25} + j\frac{24}{25}$$

d) We use the distributive property to multiply the numbers.

$$z = \left(1 + j\sqrt{3}\right)\left(\frac{\sqrt{3}}{4} - j\frac{1}{4}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} - j\frac{1}{4} + j\sqrt{3}\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2} + j\frac{1}{2}$$