

Ex: Give numerical answers to each of the following questions:

- a) Rationalize $\frac{4375 - j15,000}{7 + j24}$. Express your answer in rectangular form.
- b) Find the magnitude of $\frac{1}{2} + j\frac{\sqrt{3}}{2}$.
- c) Find the real part of $\frac{(1 + j)^4}{1 + j\sqrt{3}}$.

SOL'N: a) To rationalize, we multiply the numerator and denominator by the conjugate of the denominator.

$$\frac{4375 - j15,000}{7 + j24} \cdot \frac{7 - j24}{7 - j24} = \frac{(4375 - j15,000)(7 - j24)}{7^2 + 24^2}$$

or

$$\frac{4375 - j15,000}{7 + j24} = (7 - j24)(7 - j24) = 49 - 576 - j2(7)24$$

or

$$\frac{4375 - j15,000}{7 + j24} = -527 - j336$$

b) We think of the complex number as a vector and find its length using the Pythagorean theorem.

$$\left| \frac{1}{2} + j\frac{\sqrt{3}}{2} \right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

c)

$$\operatorname{Re} \left[\frac{(1 + j)^4}{1 + j\sqrt{3}} \right] = \operatorname{Re} \left[\frac{(\sqrt{2}e^{j45^\circ})^4}{2e^{j60^\circ}} \right] = \operatorname{Re} \left[\frac{4e^{j180^\circ}}{2e^{j60^\circ}} \right] = \operatorname{Re} [2e^{j(180^\circ - 60^\circ)}]$$

or

$$\operatorname{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = \operatorname{Re}\left[2e^{j120^\circ}\right] = \operatorname{Re}[2\cos(120^\circ) + j2\sin(120^\circ)]$$

or

$$\operatorname{Re}\left[\frac{(1+j)^4}{1+j\sqrt{3}}\right] = 2\cos(120^\circ) = 2\left(-\frac{1}{2}\right) = -1$$