Ex: a) Find the real part of $z = e^{j\pi/4}$.

b) Find the rectangular form of $e^{j\pi/3}$.

c) Find the rectangular form of $5\angle 25° \cdot 8\angle 35°$

d) Find the magnitude of $\left(\dfrac{j^{-1}}{3+j4}\right)\left(\dfrac{10e^{-j15°}}{(1+j)^2}\right)$.

e) Find the polar (magnitude and angle) form of $\sqrt{2+\sqrt{3}} - j\sqrt{2-\sqrt{3}}$

SOL'N: a) Use Euler's formula:

$$\mathrm{Re}\left[e^{j\pi/4}\right] = \mathrm{Re}\left[\cos\pi/4 + j\sin\pi/4\right] = \mathrm{Re}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}}$$

b) From the answer to (a), we have

$$e^{j\pi/3} = \cos(\pi/3) + j\sin(\pi/3) = \cos(60°) + j\sin(60°) = \frac{1}{2} + j\frac{\sqrt{3}}{2}.$$

c) We first multiply the numbers in polar form.

$$5\angle 25° \cdot 8\angle 35° = 5(8)\angle 25° + 35° = 40\angle 60° = 40e^{j60°}$$

Now we convert to rectangular form using Euler's formula.

$$5\angle 25° \cdot 8\angle 35° = 40\cos(60°) + j40\sin(60°) = 40 \cdot \frac{1}{2} + j40\frac{\sqrt{3}}{2}$$

or

$$5\angle 25° \cdot 8\angle 35° = 20 + j20\sqrt{3}$$

d) We take the magnitude of each term of a product or quotient. We must keep sums as is, however.

$$\left|\left(\frac{j^{-1}}{3+j4}\right)\left(\frac{10e^{-j15°}}{(1+j)^2}\right)\right| = \frac{\left|j^{-1}\right|}{|3+j4|}\frac{|10|\left|e^{-j15°}\right|}{|1+j|^2} = \frac{1}{\sqrt{3^2+4^2}} \cdot \frac{10 \cdot 1}{\left(\sqrt{1^2+1^2}\right)^2}$$

or

$$\left|\left(\frac{j^{-1}}{3+j4}\right)\left(\frac{10e^{-j15°}}{(1+j)^2}\right)\right| = \frac{1}{5}\cdot\frac{10\cdot 1}{\left(\sqrt{2}\right)^2} = 1$$

e) We use the Pythagorean theorem to find the magnitude:

$$A = \sqrt{2+\sqrt{3}}^{\,2} + \sqrt{2-\sqrt{3}}^{\,2} = 2+\sqrt{3}+2-\sqrt{3} = 4$$

The tangent of the angle is the imaginary part over the real part.

$$\phi = \tan^{-1}\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = 15°$$

Our answer:

$$\sqrt{2+\sqrt{3}} - j\sqrt{2-\sqrt{3}} = 4\angle 15°$$