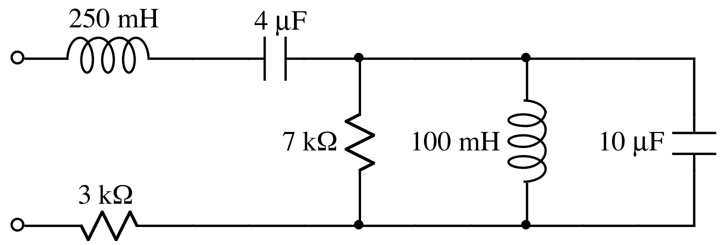
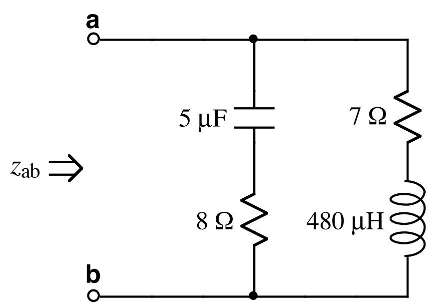
# HOMEWORK #17 prob 2 solution

Ex: a) Find the total impedance of the circuitry shown below if $\omega = 1000$ rad/s.




b) Given $\omega = 50$ k rad/s, find $z_{ab}$.




**SOL'N:** a) We convert to the frequency-domain by computing impedances.

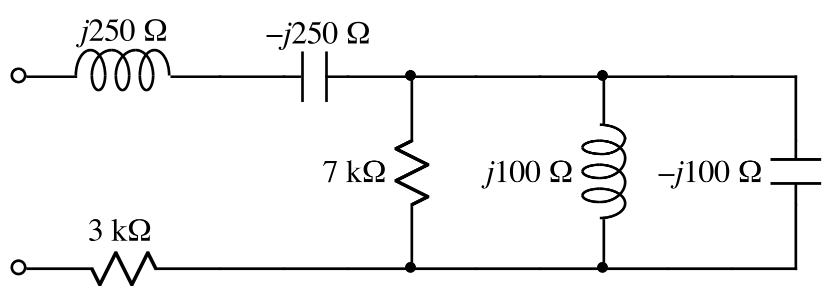
$$j\omega L = j1\text{k} \cdot 250\text{m } \Omega = j250 \text{ k}\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j1\text{k} \cdot 4\mu} \Omega = -j250 \ \Omega$$

$$j\omega L = j1\text{k} \cdot 100\text{m } \Omega = j100 \text{ k}\Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j1\text{k} \cdot 10\mu} \Omega = -j100 \ \Omega$$

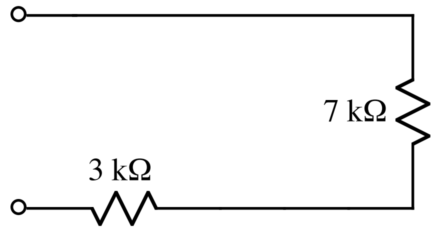
The circuit diagram in the frequency-domain is shown below.




The series $L$ and $C$ in series at the top left of the circuit sum to zero, which means they cancel out to act like a wire. The parallel $L$ and $C$ at the right combine to create an equivalent impedance of infinity, or an open circuit.

$$j100 \,\|\, -j100\ \Omega = j100\ \Omega \cdot 1 \,\|\, -1 = j100\ \Omega \cdot \frac{1(-1)}{1-1} = j100\ \Omega \cdot \frac{1}{0} = \infty\ \Omega$$

Thus, the $L$ and $C$ on the right disappear. We are left with a simple circuit consisting of only two resistors:




The equivalent impedance is obviously 10 kΩ. All the imaginary values cancelled out.

$z_{tot} = 10\ \text{k}\Omega$

soln: We compute impedances using:

$$z_R = R, \quad z_L = j\omega L, \quad z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}.$$

$$\therefore z_L = j \cdot 50\text{k rad/s} \cdot 480\ \mu\text{H}$$

$$" = j\ 50 \cdot 480\ \text{m}\Omega$$

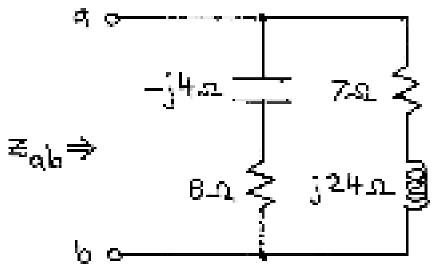
$$" = j\ 24\ \Omega$$

$$z_C = \frac{-j}{50\text{k rad/s} \cdot 5\ \mu\text{F}}$$

$$= \frac{-j}{250\text{m}}\ \Omega$$

$$= -j4\ \Omega$$

Now we draw the frequency- (or s-domain) model:




We have $Z_{ab} = (8 - j4\,\Omega) \| (7 + j24\,\Omega)$

$$= \frac{(8-j4)(7+j24)}{8-j4+7+j24}\,\Omega$$

$$= \frac{\sqrt{8^2+4^2}\ \tan^{-1}(-4/8)\ \sqrt{7^2+24^2}\ \tan^{-1}(24/7)}{15+j20}\,\Omega$$

$$= \frac{4\sqrt{5}\,\angle -26.6^\circ \cdot 25\,\angle 73.7^\circ}{25\,\angle 53.1^\circ}\,\Omega$$

$$= 4\sqrt{5}\,\angle -26.6^\circ + 73.7^\circ - 53.1^\circ\,\Omega$$

$$Z_{ab} = 4\sqrt{5}\,\angle -6^\circ\,\Omega$$