HOMEWORK #1 prob 1 solution

EX: a) Solve the following simultaneous equations for i_1 and i_2:

$$2i_1 - 9i_2 = 26.5 \qquad \frac{i_1 + i_2}{5} + 4i_1 = 7.9$$

b) Find *exact* positive values of ω satisfying the following equation given the following values: $R = 30\ \Omega$, $L = 10$ mH, and $C = 25\ \mu$F:

$$\frac{1}{\sqrt{1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}}$$

SOL'N: a) We first decide which equation is simpler in terms of solving for one variable in terms of the other. In the present case, the first equation is simpler.

$$2i_1 - 9i_2 = 26.5$$

or

$$2i_1 = 26.5 + 9i_2.$$

or

$$i_1 = \frac{26.5 + 9i_2}{2} = 13.25 + 4.5i_2.$$

We could also have solved for i_2; either choice will work. In the present case, we substitute for i_1 in the second equation. First, however, we collect the i_1 terms in the second equation.

$$i_1\left(\frac{1}{5} + 4\right) + i_2\frac{1}{5} = 7.9$$

Now we substitute for i_1:

$$(13.25 + 4.5i_2)\left(\frac{1}{5} + 4\right) + i_2\frac{1}{5} = 7.9$$

We collect i_2 terms now so that we can solve for i_2.

$$i_2\left[(4.5)\left(\frac{21}{5}\right) + \frac{1}{5}\right] + 13.25\left(\frac{21}{5}\right) = 7.9$$

or, if we multiply both sides by 10 to clear fractions

$$i_2[9(21) + 2] + 26.5(21) = 79$$

or

$$i_2 = \frac{79 - 26.5(21)}{9(21) + 2} = -2.5$$

Now that i_2 is known, the equation derived earlier for i_1 in terms of i_2 yields the value of i_1:

$$i_1 = 13.25 - 4.5(2.5) = 2$$

b) When doing algebra, any operation that affects both sides the same way without creating a divide by zero is allowed. Thus, inverting both sides is allowed and yields the following result:

$$\sqrt{1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}$$

Squaring both sides eliminates the square roots:

$$1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2 = 2$$

Subtracting unity from both sides yields an equation with perfect squares on both sides:

$$\frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2 = 1$$

Taking the square root of both sides yields a simpler problem, but both the positive and negative square roots must be taken into account:

$$\sqrt{\frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{1}$$

or, more precisely,

$$\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right) = \pm 1 \ .$$

This is a pair of quadratic equations, which becomes apparent after multiplying both sides by ω:

$$\frac{1}{R}\left(\omega^2 L-\frac{1}{C}\right)=\pm\omega \ .$$

The remaining steps are straightforward algebra:

$$\left(\omega^2 L-\frac{1}{C}\right)=\pm R\omega$$

or

$$\omega^2 L \pm R\omega-\frac{1}{C}=0$$

or, if the coefficient of the squared term is set to unity:

$$\omega^2 \pm \frac{R}{L}\omega-\frac{1}{LC}=0$$

This is two quadratic equations, with four roots:

$$\omega=\pm\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$$

Now the challenge is to determine which roots are positive. Examination of the radical (i.e., square root) term, however, reveals that, because *R*, *L*, and *C* are positive, its magnitude is always larger than $R/2L$, meaning it must added in order to obtain a positive value.

$$\omega_1=-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$$

$$\omega_2=+\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$$