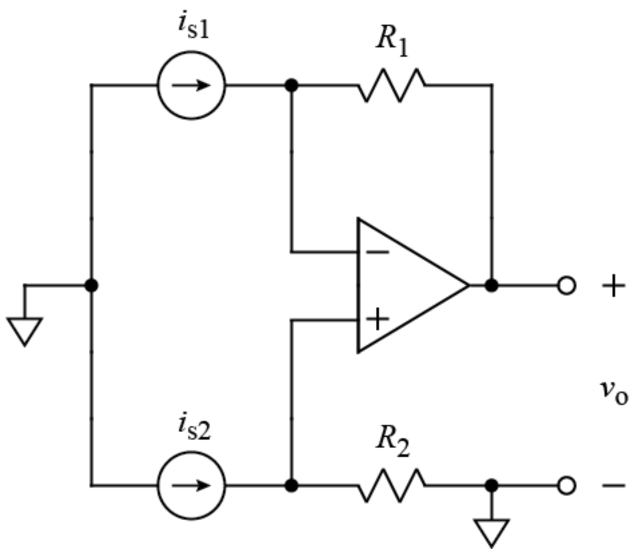
Homework 20

1.



a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_{s1}, i_{s2}, R_1, and R_2.

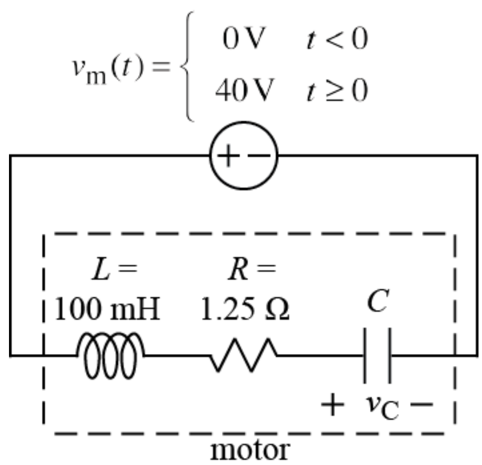
b) Derive a symbolic expression for v_o in terms of common mode and differential input currents:

$$i_{cm} \equiv \frac{(i_{s2} + i_{s1})}{2} \quad \text{and} \quad i_{dm} \equiv i_{s2} - i_{s1}$$

The expression must contain not more than the parameters i_{cm}, i_{dm}, R_1, and R_2. Write the expression as i_{cm} times a term plus i_{dm} times a term. Hint: start by writing i_{s1} and i_{s2} in terms of i_{cm} and i_{dm}.

c) What condition must be satisfied in order for the above circuit to amplify only i_{dm}?

2.



The above circuit is a model of a motor driven by a voltage source. The inductor and resistor represent the inductance and resistance of the windings of the motor, and the capacitor models the back emf (voltage) generated by the windings when the motor is rotating. (When the motor rotates, the windings experience a changing magnetic field that creates a voltage drop in the windings. Thus, the capacitor in this model is considered to be part of the windings, along with the inductor and resistor.) The capacitor value is related to motor parameters:

$$C = \frac{J}{K^2}$$

where $J \equiv$ moment of inertia of motor

$K \equiv$ emf or torque constant of motor

The motor is at rest initially. After being off (0 V) for a long time, the voltage source steps to 40 V at $t = 0$.

a) Find the value of C given the following characteristic roots for the *RLC* model of the motor:

$$s_1 = -1.25 \text{ r/s}, \;\; s_2 = -11.25 \text{ r/s}$$

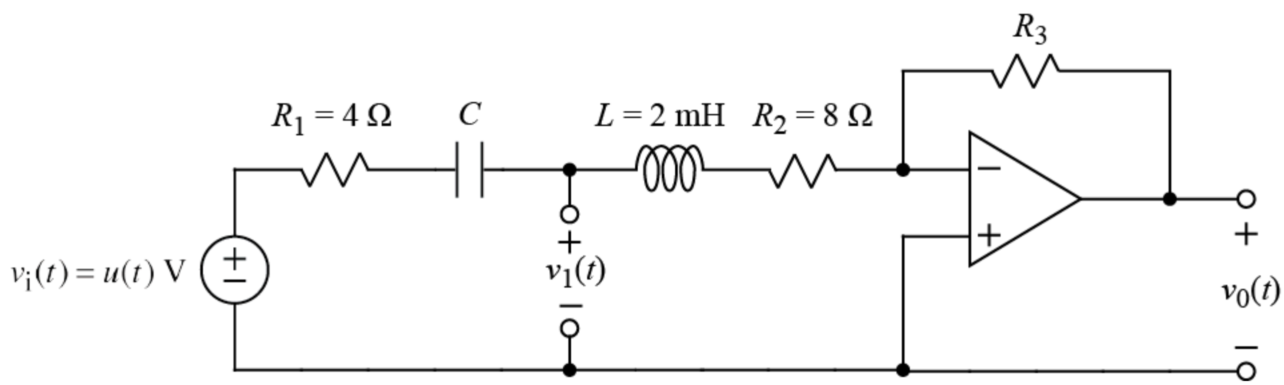
b) Using your C value from (a), find a numerical expression for the back emf, $v_C(t)$, for $t > 0$.

c) The angular velocity of the motor rotation is related to the voltage across C by the following equation:

$$\omega(t) = \frac{v_C(t)}{K}$$

If $J = 10/9 \approx 1.111$ in SI units, find the velocity of the motor in rotations per second at time $t = 1$ s.

3.



The voltage source in the above circuit is off (0 V) for $t < 0$.

An engineer wishes to use the above circuit to create two decaying sinusoidal signals 120° out-of-phase to drive a three-phase motor for a short time. (A third signal that is 120° out-of-phase with the first two may be created by an additional op-amp circuit, not shown, that computes $-v_0 - v_1$.) The signal at $v_0(t)$ will necessarily be a decaying sinusoid of the following form:

$$v_0(t) = -v_m e^{-\alpha t} \sin(\beta t)$$

where v_m, α, and β are positive real-valued constants.

The design problem now is to create a $v_1(t)$ signal that is 120° out-of-phase with $v_0(t)$.

a) Find a symbolic expression for the Laplace-transformed output, $\mathbf{V}_1(s)$, in terms of not more than R_1, R_2, R_3, L, C, and values of sources or constants.

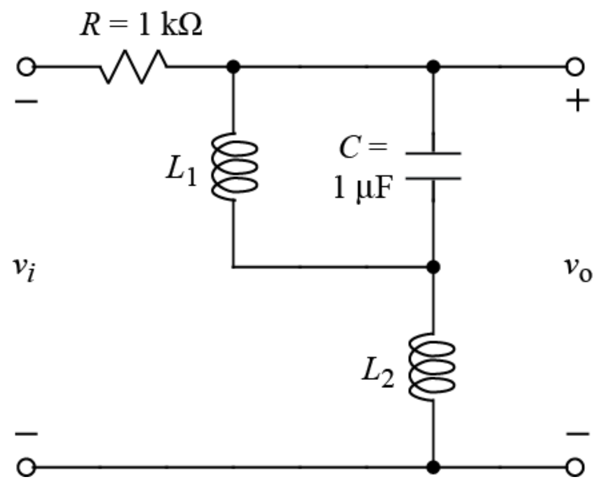
b) Choose a numerical value for C to make

$$v_1(t) = v_m e^{-\alpha t} \cos(\beta t - 30°).$$

Hint: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

c) Why could the desired $v_1(t)$ not be obtained if the positions of the L and C were reversed?

4.



The above filter circuit is intended to be complimentary to the filter designed in ECE 2240 Lab 4. That is, it is designed to have low gain at frequencies where the filter in Lab 4 had high gain, and vice versa.

a) Find values of $L_1 \neq 0$ and $L_2 \neq 0$ such that the magnitude of the filter's transfer function, H, has the following values:

$$|H(j\omega)| = 1 \ \text{ at } \ \omega = 2\pi \cdot 1\ \text{kHz}$$

$$|H(j\omega)| = 0 \ \text{ at } \ \omega = 2\pi \cdot 3\ \text{kHz}$$

b) Using the values of L_1 and L_2 you found in part (a), find the approximate magnitude of $H(j\omega)$ at $\omega = 2\pi \cdot 5$ kr/s. Your answer should be within 15% of the actual value.

c) Describe how increasing the value of R would affect the shape of the plot of the gain, $|H(j\omega)|$, versus ω.

5. Make up a problem for the final exam and solve it.