Ex:


For the circuit shown, write three independent equations for the node voltages $v_{1}, v_{2}$, and $v_{3}$. The quantity $i_{\mathrm{x}}$ must not appear in the equations.

Sol'n: For the $v_{1}$ node on the left:

$$
\frac{v_{1}+v_{\mathrm{s} 1}-v_{2}}{R_{1}+R_{3}}+\alpha \frac{v_{1}+v_{\mathrm{s} 1}-v_{2}}{R_{1}+R_{3}}+\frac{v_{1}-v_{3}}{R_{2}}=0 \mathrm{~A}
$$

Note that $i_{\mathrm{x}}$ is the same as the current in the top branch from $v_{1}$ to $v_{2}$. We substitute this current for $i_{\mathrm{x}}$ in the middle term.

Nodes $v_{2}$ and $v_{3}$ form a super-node. The voltage equation for the nodes is

$$
v_{2}+v_{\mathrm{s} 2}=v_{3} .
$$

The current summation for the super-node is formed by all currents flowing out of the $v_{2}$ and $v_{3}$ nodes except those flowing in $v_{\mathrm{s} 2}$ :

$$
\frac{v_{2}-v_{\mathrm{s} 1}-v_{1}}{R_{1}+R_{3}}+\alpha \frac{v_{2}-v_{\mathrm{s} 1}-v_{1}}{R_{1}+R_{3}}+\frac{v_{2}}{R_{4}}+\frac{v_{3}-v_{1}}{R_{2}}-i_{\mathrm{s}}=0 \mathrm{~A} .
$$

This completes the set of three equations.

