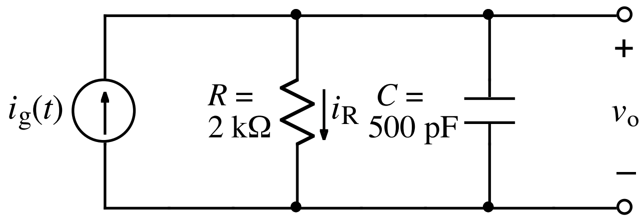
EX: After being zero for a long time, the value of $i_g(t)$ changes to 15 mA at t = 0 (and remains at 15 mA as time increases to infinity).



a) Find an expression for $v_o(t)$ for $t > 0$.
b) Find the current, i_R, in R as a function of time.

SOL'N: a) The following general form of solution applies to any RC circuit with a single capacitor:

$$v_C(t \geq 0) = v_C(t \to \infty) + [v_C(t = 0^+) - v_C(t \to \infty)]e^{-t/R_{\text{Th}}C}$$

The Thevenin resistance, R_{Th}, is for the circuit after t = 0 (with the C removed) as seen from the terminals where the C is connected. In the present case, we have $R_{\text{Th}} = 2\ \text{k}\Omega$.

$$R_{\text{Th}}C = 2\ \text{k}\Omega \cdot 500\ \text{pF} = 1\ \mu\text{s}$$

For time $t = 0^-$, the current source will be off and the capacitor will have discharged to zero volts. Since the voltage on the capacitor is an energy variable, it will not change instantly. Thus, the initial capacitor voltage is zero.

$$v_C(t = 0^+) = 0\ \text{V}$$

For time approaching infinity, the capacitor will charge to a final value and no current will flow in the capacitor. Thus, the capacitor will act like an open circuit. It follows that all the current from the source, $i_g(t)$, will flow through the resistor, resulting in a voltage across the resistor (and capacitor) of $i_g R$.

$$v_C(t \to \infty) = i_g R = 15\ \text{mA} \cdot 2\ \text{k}\Omega = 30\ \text{V}$$

Substituting values, we have the following result:

$$v_C(t \geq 0) = 30\ \text{V} + [0\ \text{V} - 30\ \text{V}]e^{-t/20\mu\text{s}} = 30\ \text{V} - 30\ \text{V} \cdot e^{-t/1\mu\text{s}}$$

b) The following general form of solution applies to any current in any RC circuit with a single capacitor:

$$i(t \geq 0) = i(t \to \infty) + [i(t = 0^+) - i(t \to \infty)]e^{-t/R_{\text{Th}}C}$$

In the present case, this applies to the resistor current:

$$i_R(t \geq 0) = i_R(t \to \infty) + [i_R(t = 0^+) - i_R(t \to \infty)]e^{-t/R_{\text{Th}}C}$$

We have the same Thevenin resistance and time constant as before:

$$R_{\text{Th}}C = 1\ \mu\text{s}$$

The initial value of i_R comes from time $t = 0^+$. At that moment in time, the voltage across the C is 0 V. This voltage appears across R and, by Ohm's law, no current will flow in R.

$$i_R(t = 0^+) = 0\ \text{A}$$

As time approaches infinity, the voltage across the R is the final voltage across the C, which is 30 V. By Ohm's law, the current through the R will be this voltage divided by R:

$$i_R(t \to \infty) = 30\ \text{V}/2\ \text{k}\Omega = 15\ \text{mA}$$

Substituting values, we have the following result:

$$i_R(t \geq 0) = 15\ \text{mA} + [0 - 15\ \text{mA}]e^{-t/1\mu\text{s}} = 15\ \text{mA} - 15\ \text{mA} \cdot e^{-t/1\mu\text{s}}$$