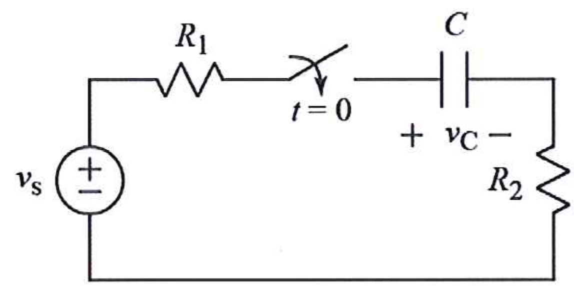
# HOMEWORK #9 prob 1 solution

**Ex:**




$R_1 = 1.5\ \text{M}\Omega \qquad R_2 = 1\ \text{M}\Omega \qquad C = 3.3\ \text{nF} \qquad v_s = 2.3\ \text{V}$

After being open for a long time, the switch closes at $t = 0$. The initial voltage on the capacitor is $v_C(t = 0^+) = 1.5$ V. Hint: think Thevenin equivalent for the circuit the capacitor is connected to.

a) Find an expression for $v_C(t)$ for $t \geq 0$.

b) Find the energy stored in the capacitor as $t$ approaches infinity.

sol'n: a) We use the general form of solution:

$$v_c(t \geq 0) = v_c(t \to \infty) + [v_c(0^+) - v_c(t \to \infty)]e^{-t/R_{Th}C}$$

Now we find the key quantities, $v_c(0^+)$, $R_{Th}$ (for $t > 0$), and $v_c(t \to \infty)$.

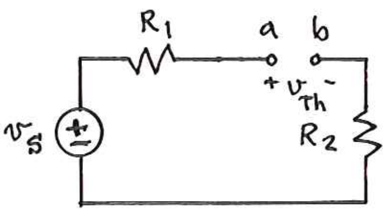
To find $v_c(0^+)$, we observe that $v_c(0^+) = v_c(0^-)$ since $v_c$ is an energy variable that cannot change instantly. This is helpful because, at $t = 0^-$, the circuit has been sitting for a long time and has settled to constant i's and v's. This in turn implies $dv_c(t)/dt = 0$. This in turn implies $i_c(t) = 0$. This in turn implies that the C acts like an open circuit.

Here, the switch is open and the C acts like an open. Using a v-loop, we find that there is no current in the R's and no $v$-drop across the R's, meaning the $v_s$ is dropped across the switch and C. There is no way to determine how much of $v_s$ is dropped across the switch and how much is dropped across C. So we need more information.

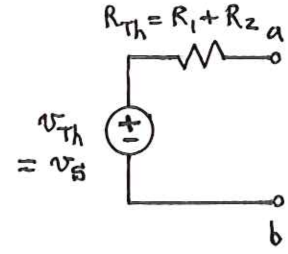
Fortunately, the problem states that the voltage across C is 1.5 V at $t=0^+$.

Next, we remove the C from the circuit and find the Thevenin equivalent of the remaining circuit. (The terminals a, b for the Thevenin equivalent are where the C is connected.)

Our circuit for the Thevenin equivalent for $t>0$:




Since no current flows, $v_{Th}$ (= $v_{a,b}$ in above circuit) will be $v_s$. $R_{Th}$ is the R seen looking into a, b with $v_s$ set to zero, which is $R_{Th} = R_1 + R_2$.




Our time constant is $\tau = R_{Th} C = (R_1 + R_2) C$

$$\tau = (1.5\ M\Omega + 1\ M\Omega)\, 3.3\,nF = 8.25\ ms$$

For $t \to \infty$, our circuit model is the above Thevenin equivalent with $C$ attached to a, b. The $C$, however, acts like an open, so $v_C = v_{Th}$.

$$v_C(t \to \infty) = v_{Th} = v_S = 2.3V$$

We plug values into the general soln:

$$v_C(t \geq 0) = 2.3V + [1.5V - 2.3V]e^{-t/8.25ms}$$

or

$$v_C(t \geq 0) = 2.3V - 0.8V e^{-t/8.25 ms}$$

b) Stored energy is $w_C = \frac{1}{2} C v_C^2$.

$$w_C(t \to \infty) = \frac{1}{2}(3.3nF)(2.3V)^2$$

or

$$w_C(t \to \infty) \doteq 8.73 \ nJ$$