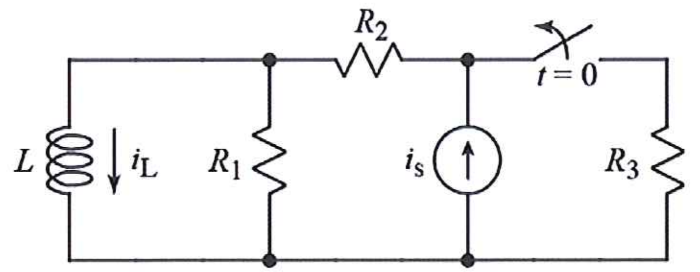
HOMEWORK #9 prob 2 solution

Ex:



$R_1 = 12\ \text{k}\Omega \quad R_2 = 24\ \text{k}\Omega \quad R_3 = 36\ \text{k}\Omega \quad L = 5\ \text{nH} \quad i_s = 6\ \text{mA}$

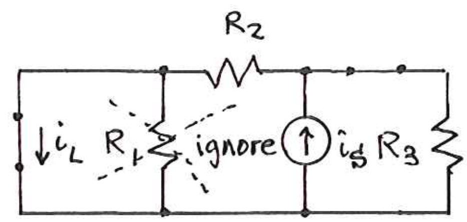
After being closed for a long time, the switch opens at $t = 0$.

a) Find $i_L(t = 0^+)$.

b) Find $i_L(t)$ for $t > 0$.

c) Find the time t when $i_L(t) = 4.5$ mA.

sol'n: a) $t = 0^-$: switch is closed, L acts like wire



The wire for L bypasses R_1 with a short circuit, allowing us to ignore R_1.

We have a current divider.

$$i_L(0^-) = i_s \frac{\frac{1}{R_2}}{\frac{1}{R_2} + \frac{1}{R_3}} = i_s \frac{R_3}{R_2 + R_3}$$

$$i_L(0^-) = 6\ \text{mA} \cdot \frac{36\ \text{k}\Omega}{24\ \text{k}\Omega + 36\ \text{k}\Omega} = 3.6\ \text{mA}$$

b) We use the general form of sol'n:

$$i_L(t>0) = i_L(t\to\infty) + [i_L(0^+) - i_L(t\to\infty)]e^{-t/\tau}$$

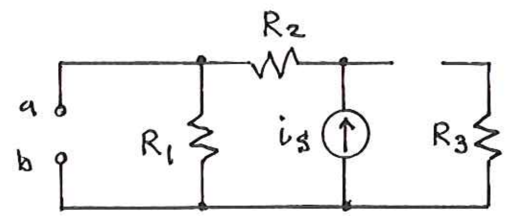
where $\tau = L/R_{Th}$.

Since i_L is an energy variable, (as in $w_L = \frac{1}{2}Li_L^2$), it cannot change instantly:

$$i_L(0^+) = i_L(0^-) = 3.6 \text{ mA}$$

Now we find the Thevenin equivalent of the circuit as seen by the L and use that to find R_{Th} and $i_L(t\to\infty) = i_{sc}$ where $i_{sc} \equiv$ short circuit current for the Thevenin equivalent.

Our circuit for which we are looking for the Thevenin equivalent:



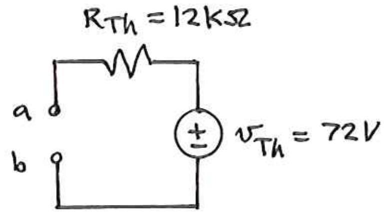
The switch is open since $t > 0$.

$v_{Th} = v_{a,b} = i_s R_1$ since all of i_s flows through R_1.

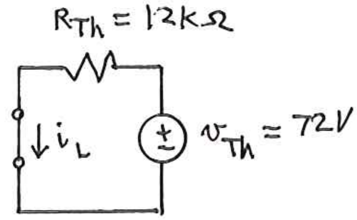
To find R_{Th}, we turn off i_s, which makes it an open circuit. Then we look in from a,b. R_2 and R_3 dangle and may be ignored.

$$R_{Th} = R_1$$

Using numbers: $v_{Th} = 6\text{mA} \cdot 12\text{k}\Omega = 72\text{V}$, $R_{Th} = 12\text{k}\Omega$



If we consider $t \to \infty$, the L connected across a, b will act like a wire.



$$i_L(t \to \infty) = v_{Th}/R_{Th} = \frac{72V}{12k\Omega} = 6\text{ mA}$$

Our time constant is

$$\tau = \frac{L}{R_{Th}} = \frac{5\text{ nH}}{12k\Omega} \doteq 0.417\text{ ps}$$

Putting the values into the general solution yields the following answer:

$$i_L(t>0) = 6\text{ mA} + (3.6\text{mA} - 6\text{mA})\,e^{-t/0.417\text{ps}}$$

or

$$i_L(t>0) = 6\text{mA} - 2.4\text{mA}\,e^{-t/0.417\text{ps}}$$

c) To find the time t when $i_L = 4.5\text{mA}$, we use the answer to (b) and isolate the exponential term so we can take the natural log of both sides.

$$4.5\text{ mA} = 6\text{mA} - 2.4\text{mA}\,e^{-t/0.417\text{ps}}$$

or

$$4.5\text{mA} - 6\text{mA} = -2.4\text{mA}\,e^{-t/0.417\text{ps}}$$

or

$$-1.5\ mA = -2.4\ mA\ e^{-t/0.417\,ps}$$

or

$$1.5\,mA = 2.4\,mA\,e^{-t/0.417\,ps}$$

or

$$\frac{1.5\,mA}{2.4\,mA} = e^{-t/0.417\,ps}$$

or

$$\frac{5}{8} = e^{-t/0.417\,ps}$$

or

$$\ln\left(\frac{5}{8}\right) = -t/0.417\,ps$$

or

$$-t = 0.417\,ps \cdot \ln\left(\frac{5}{8}\right)$$

or

$$t = 0.417\,ps \cdot \ln\left(\frac{8}{5}\right) = 0.417\,ps \cdot \ln(1.6)$$

or

$$t \doteq 0.196\ ps$$