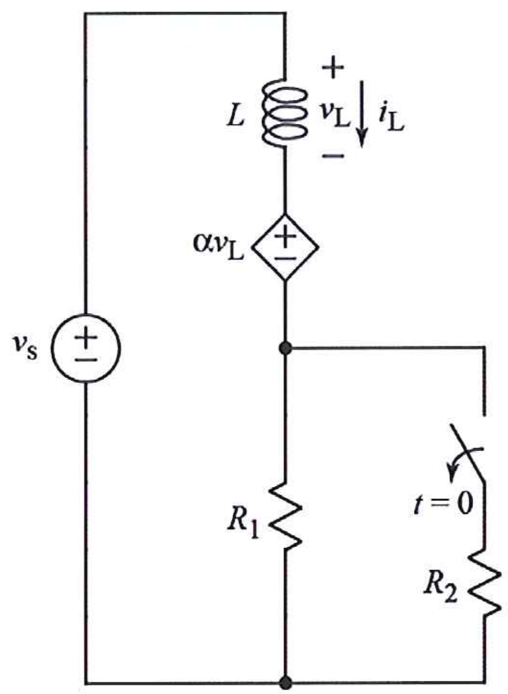
HOMEWORK #9 prob 4 solution

Ex:

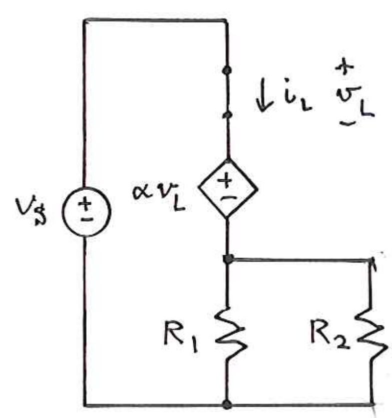


After being closed for a long time, the switch opens at $t = 0$.

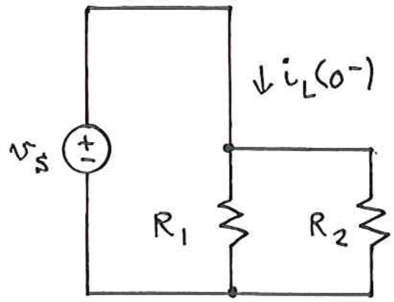
a) Find an expression for $i_L(0^-)$.

b) Find an expression for $i_L(t)$ for $t > 0$.

sol'n: a) $t = 0^-$: L = wire, switch is closed



Since the L acts like a wire, $v_L(0^-) = 0V$ and $\alpha v_L(0^-) = 0V$. So the model simplifies.



$$i_L(0^-) = \frac{v_s}{R_1 \parallel R_2}$$

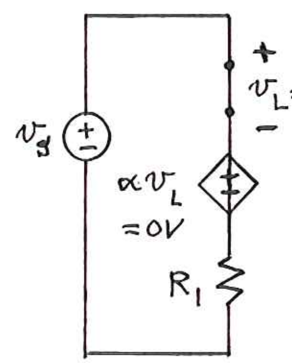
b) We use the general sol'n for RL circuits:

$$i_L(t>0) = i_L(t\to\infty) + \left[i_L(0^+) - i_L(t\to\infty)\right]e^{-t/\tau}$$

where $\tau = L/R_{Th}$ where R_{Th} is for $t>0$.

Since i_L is an energy variable, it does not change instantly. Thus, $i_L(0^+) = i_L(0^-) = \dfrac{v_s}{R_1 \parallel R_2}$.

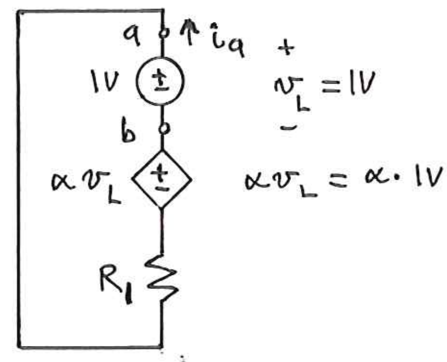
For $t\to\infty$: L = wire, switch is open



The dependent source will be 0V since $v_L = 0V$, (L = wire).

$$i_L(t\to\infty) = \frac{v_s}{R_1}$$

Finally, we find R_{Th} for $t>0$. We can find R_{Th} directly by hooking a 1V source up to the output (where the L was connected) and turning off v_s.



$$i_a = \frac{v_L + \alpha v_L}{R_1} = \frac{(1+\alpha)\ 1V}{R_1}$$

$$R_{Th} = \frac{1V}{i_a} = \frac{R_1}{1+\alpha}$$

$$\tau = \frac{L}{R_{Th}} = \frac{L}{\frac{R_1}{1+\alpha}} = L\frac{(1+\alpha)}{R_1}$$

Using the general soln, we have our answer:

$$i_L(t>0) = \frac{v_S}{R_1} + \left(\frac{v_S}{R_1 \| R_2} - \frac{v_S}{R_1}\right) e^{-t/(1+\alpha)L/R_1}$$

Note: $\frac{v_S}{R_1 \| R_2} = v_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{v_S}{R_1} = \frac{v_S}{R_2}$