1. 


a) The above circuit operates in linear mode. Derive a symbolic expression for $v_{0}$. The expression must contain not more than the parameters $v_{\mathrm{s} 1}, v_{\mathrm{s} 2}, R_{1}, R_{2}, R_{3}$, $R_{4}$, and $R_{5}$.
b) If $v_{\mathrm{s} 1}=0 \mathrm{~V}$ and $v_{\mathrm{s} 2}=1 \mathrm{~V}$, find the value of $R_{5}$ that will yield an output voltage of $v_{\mathrm{o}}=1 \mathrm{~V}$.
c) Derive a symbolic expression for $v_{\mathrm{O}}$ in terms of common mode and differential input voltages:

$$
v_{\mathrm{cm}} \equiv \frac{\left(v_{s 2}+v_{s 1}\right)}{2} \quad \text { and } \quad v_{\mathrm{dm}} \equiv v_{s 2}-v_{s 1}
$$

The expression must contain not more than the parameters $v_{\mathrm{cm}}, v_{\mathrm{dm}}, R_{1}, R_{2}, R_{3}$, $R_{4}$, and $R_{5}$. Write the expression as $v_{\mathrm{cm}}$ times a term plus $v_{\mathrm{dm}}$ times a term. Hint: start by writing $v_{\mathrm{s} 1}$ and $v_{\mathrm{s} 2}$ in terms of $v_{\mathrm{cm}}$ and $v_{\mathrm{dm}}$ :

$$
v_{s 1}=v_{\mathrm{cm}}-\frac{v_{\mathrm{dm}}}{2} \quad \text { and } \quad v_{\mathrm{s} 2}=v_{\mathrm{cm}}+\frac{v_{\mathrm{dm}}}{2}
$$

d) Find the numerical value of the circuit's input resistance, $R_{\mathrm{in}}$, as seen by source $v_{\mathrm{s} 2}$. In other words, write a formula for voltage, $v_{s 2}$, divided by $i_{2}$ :

$$
R_{\mathrm{in}} \equiv \frac{v_{s 2}}{i_{2}}
$$

2. 



After being closed for a long time, the switch opens at $t=0$.
The above circuit is an analog "one-shot" circuit that, once charged, produces a short, rounded current-pulse resembling the current that flows in a synapse of a neuron. The circuit is critically damped.
a) Find the value of $R$ that makes the circuit critically-damped.
b) Using the $R$ value from (a), find a numerical expression for the inductor current, $i(t)$, for $t>0$.
3.


The current source in the above circuit is off for $t<0$.
a) Find a symbolic expression for the Laplace-transformed output, $\mathbf{V}_{0}(s)$, in terms of not more than $R_{1}, R_{2}, L, C$, and values of sources or constants.
b) Choose a numerical value for $R_{1}$ to make

$$
v_{1}(t)=v_{m}-v_{m} e^{-\alpha t}\left[\cos (\beta t)+\frac{1}{2} \sin (\beta t)\right]
$$

where $v_{m}, \alpha$, and $\beta$ are real-valued constants.
Hint: $C$ behaves as though it is in parallel with $L$ and $R_{1}$.
4.


The above filter circuit is being considered for use in a communication system to detect whether received signals represent binary zeros or binary ones. The plan is to use an inexpensive design with rectangular waveforms (rather than sinusoids). A zero will be signaled by a square wave (not shown), and a one will be signaled by a rectangular wave having $2 / 3$ duty cycle (shown above). The filter for detecting a zero is designed to pass the fundamental frequency of the waveforms, which is the same as one over the period of the waveform shown above. The issues addressed in this problem are the design of the filter and how well it blocks the waveform representing a "one".
a) Find values of $L_{1} \neq 0$ and $L_{2} \neq 0$ such that the magnitude of the filter's transfer function, $H(j \omega)$, equals one for the fundamental frequency, $\omega_{0}$, and zero for frequency $3 \omega_{0} / 2$, (which the engineer proposing the circuit believes is present in the signal for a "one").
b) Find the numerical value of the magnitude of $H$ for frequency $2 \omega_{0}$.

