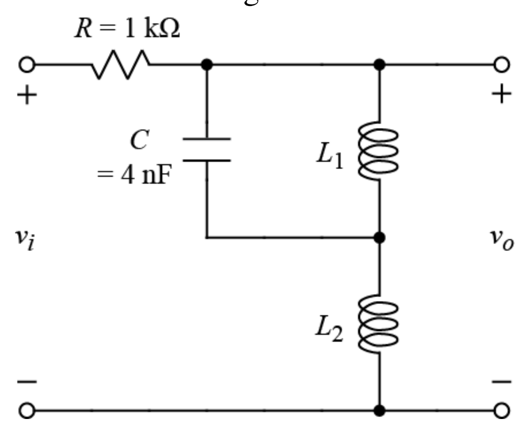
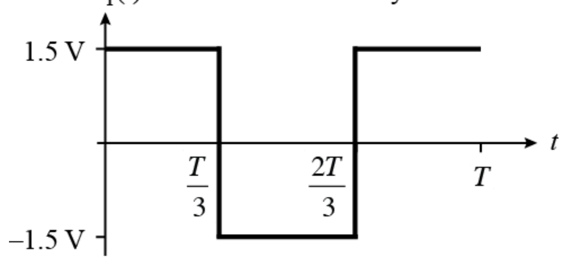
Ex: Find the value of each of the following:



$v_1(t)$ = waveform for binary 1



T = one period of $v_i(t) = 2\pi$ ns

$$v_1(t) = \begin{cases} 1.5\text{ V} & 0 \le t < T/3 \\ -1.5\text{ V} & T/3 \le t < 2T/3 \\ 1.5\text{ V} & 2T/3 \le t \le T \end{cases}$$

The above filter circuit is being considered for use in a communication system to detect whether received signals represent binary zeros or binary ones. The plan is to use an inexpensive design with rectangular waveforms (rather than sinusoids). A zero will be signaled by a square wave (not shown), and a one will be signaled by a rectangular wave having 2/3 duty cycle (shown above). The filter for detecting a zero is designed to pass the fundamental frequency of the waveforms, which is the same as one over the period of the waveform shown above. The issues addressed in this problem are the design of the filter and how well it blocks the waveform representing a "one".

a) Find values of $L_1 \neq 0$ and $L_2 \neq 0$ such that the magnitude of the filter's transfer function, $H(j\omega)$, equals one for the fundamental frequency, ω_0, and zero for frequency $3\omega_0/2$, (which the engineer proposing the circuit believes is present in the signal for a "one").

b) Find the numerical value of the magnitude of H for frequency $2\omega_o$.

sol'n a) We can achieve a gain (i.e., $|H| = 1$) by having a resonance of $L_1 \| C$ (i.e. having $j\omega L \| \frac{1}{j\omega C} = \infty$) at $\omega = \omega_0$. At resonance, we have $j\omega L + \frac{1}{j\omega C} = 0$:

$$j\omega_0 L \,\Big\|\, \frac{1}{j\omega_0 C} = \frac{j\omega_0 L / j\omega_0 C}{j\omega_0 L + 1/j\omega_0 C} = \frac{L/C}{0} = \infty$$

This gives
$$|H| = \frac{j\omega_0 L \,\|\, \frac{1}{j\omega_0 C}}{j\omega_0 L \,\|\, \frac{1}{j\omega_0 C} + R} = \frac{\infty}{\infty} = 1.$$

Since $\omega_0^2 = \frac{1}{L_1 C} = \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{2\pi n}\right)^2 = 1\,G^2 r^2/s^2$, we can solve for L_1:

$$L_1 = \frac{1}{\omega_0^2 C} = \frac{1 \;\; H}{1G^2 \, 4n} = 250\,pH$$

To achieve $|H| = 0$, we create a resonance of $j\omega L_1 \| \frac{1}{j\omega C}$ and $j\omega L_2$ so that C, L_1, and L_2 act like a wire: at $\frac{3\omega_0}{2}$

$$\frac{j\frac{3}{2}\omega_0 L_1 \cdot \frac{1}{j3\frac{\omega_0}{2}C}}{j\frac{3}{2}\omega_0 L_1 + \frac{1}{j\frac{3}{2}\omega_0 C}} + j2\omega_0 L_2 = 0$$

$$\frac{\cancel{j1.5G} \cdot 250p \cdot 1/\cancel{j1.5G}\, 4n\,\Omega}{j1.5G \cdot 250p + 1/j1.5\cancel{G}\,4\cancel{n}} + j\,2G\,L_2 = 0$$

$$\frac{1/16}{j\frac{1.5}{4} - j\frac{1}{6}} + j1.5G\,L_2 = 0$$

$$\frac{3}{j18 - j8} + j1.5G L_2 = 0$$

$$1.5G L_2 = \frac{3}{10}$$

$$L_2 = \frac{3}{15G} = 200 \text{ pF}$$

b) From part (a) we have the value of the fundamental frequency, from which we may compute $2\omega_o$.

$$2\omega_o = 2\,\text{Gr/s}$$

The filter transfer function is the ratio of impedances.

$$H(j\omega) = \frac{z_2}{z_1 + z_2}$$

where

$$z_1 = R = 1\text{k}\Omega$$

and

$$z_2 = j\omega L_2 + \frac{1}{\frac{1}{j\omega L_1} + j\omega C}$$

or

$$z_2 = j2\text{G} \cdot 200\text{p} + \frac{1}{\frac{1}{j2\text{G} \cdot 250\text{p}} + j2\text{G} \cdot 4\text{n}}\ \Omega$$

or

$$z_2 = j0.4 + \frac{1}{\frac{1}{j500\text{m}} + j8}\ \Omega = j0.4 + \frac{1}{-j2 + j8}\ \Omega$$

or

$$z_2 = j0.4 + \frac{1}{j6}\ \Omega = j0.4 - j0.167\ \Omega = j0.233\ \Omega$$

Plugging in the numbers, we compute the magnitude of *H*.

$$\left|H(j\omega)\right| \approx \left|\frac{z_2}{z_1 + z_2}\right| \approx \left|\frac{j0.233}{1\text{k} + j0.233}\right| \approx 0.233\text{m}$$