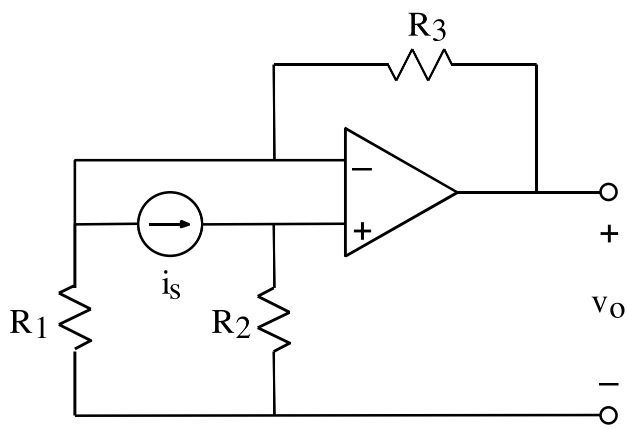
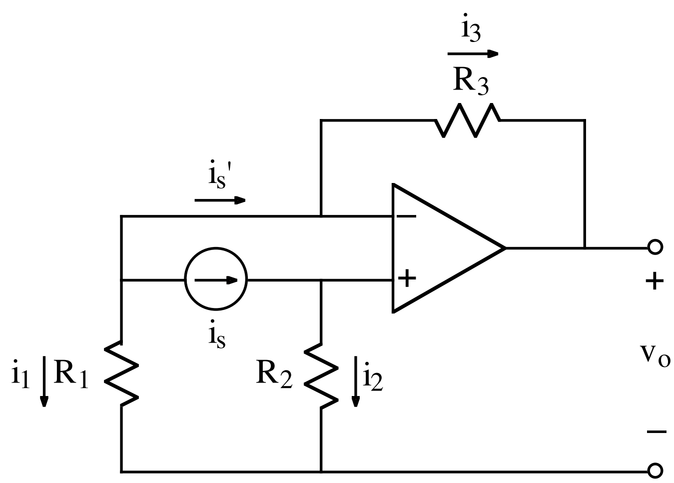
4. (30 points)

The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than i_s, R_1, R_2, and R_3.



ans: $v_o = i_s(R_2 + R_3 + \frac{R_2 R_3}{R_1})$

sol'n: Using passive sign convention, label voltage drop and current measurement polarities.



Assume an ideal op-amp since we are in linear mode and we are solving for DC conditions.

Use the standard procedure to find v_o:

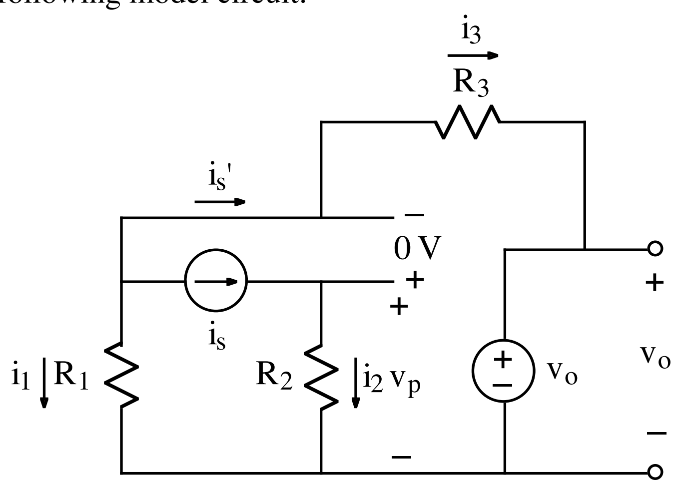
1) Calculate the voltage drop, v_p, from the – input of the op amp to the bottom wire, (i.e. across R_2). We use Kirchhoff's law for summation of currents out of the node next to the + terminal of the op-amp. Since $i_p = 0$, (i.e., no current flows into the + terminal), we conclude that

$$i_2 = i_s.$$

By Ohm's law, it follows that

$$v_p = i_s R_2$$

2) Remove the op amp, put a source called v_o across the terminals where we measure the output, and assume a 0V drop from + input to – input of op amp. (Because of negative feedback, the op-amp output voltage, v_o, reaches an equilibrium level that results in a nearly 0V drop from + input to – input.) Now we have the following model circuit:



Note that we model the – terminal of the op-amp as a voltage source with zero current flowing into it. We add this constraint on the current with the understanding that v_o must have a value that results in $i_n = 0$.

3) Calculate i_s', the current flowing toward the – terminal from the left. Note that current i_s' has a name similar to the current source only by coincidence.) Now we observe that the voltage drop from the – terminal of the op amp to the bottom wire is $v_p - 0V = v_p$. It is as though we have a voltage source of voltage v_p from the – terminal of the op amp to the bottom wire. This voltage source separates the two sides of the circuit. What happens on the right side of the circuit will not affect the value of i_s'. Thus, we utilize the left side of the circuit to calculate i_s'.

We use a summation of node currents at the node above R_1 to calculate the value of i_s':

$$i_s' + i_1 + i_s = 0 \qquad \text{or} \qquad i_s' = -(i_1 + i_s)$$

But we can also calculate i_1 because we have v_n across R_1:

$$i_1 = v_p/R_1 = i_sR_2/R_1$$

We can now solve for i_s':

$$i_s' = -i_s(R_2/R_1 + 1) = -i_s(1 + R_2/R_1)$$

4) Calculate i_3, the current flowing in the negative feedback path on the right side of the circuit. Again, it is as though we had a voltage source of voltage v_p from the – terminal of the op amp to the bottom wire. We utilize this voltage source and a voltage loop around the right side of the circuit to calculate v_o:

$$v_p - i_3R_3 - v_o = 0 \quad \text{or} \quad i_3 = -(v_o - v_p)/R_3 = -(v_o - i_sR_2)/R_3$$

5) Since $i_n = 0$, set $i_s' = i_3$.

$$i_s' = -i_s(1 + R_2/R_1) = i_3 = -(v_o - i_sR_2)/R_3$$

Equate the 2nd and 4th expressions and solve for v_o.

$$i_s(1 + R_2/R_1) = (v_o - i_sR_2)/R_3 \quad \text{or} \quad v_o = i_sR_3(1 + R_2/R_1) + i_sR_2 \quad \text{or}$$

$$v_o = i_s(R_2 + R_3 + R_2R_3/R_1)$$

Consistency checks:

1) By linearity and fact that we have one source, i_s, we expect v_o to be directly proportional to i_s. Agrees with our formula for v_o. ✓

2) Consider $R_2 = 0$: Then $v_p = 0$. Therefore, $i_1 = 0$ and $i_3 = -i_s$. Then, $v_o = -i_3R_3 = i_sR_3$. Agrees with our formula for v_o when we plug in $R_2 = 0$. ✓

3) Consider $R_1 = \infty$ (open circuit): $v_p = i_sR_2$, $i_3 = -i_s$, and $v_o = v_n - i_3R_3 = i_sR_2 + i_sR_3$. Agrees with our formula for v_o when we plug in $R_1 = \infty$. ✓

4) Consider $R_3 = 0$: Then $v_o = v_n = v_p = i_sR_2$. Agrees with our formula for v_o when we plug in $R_3 = 0$. ✓