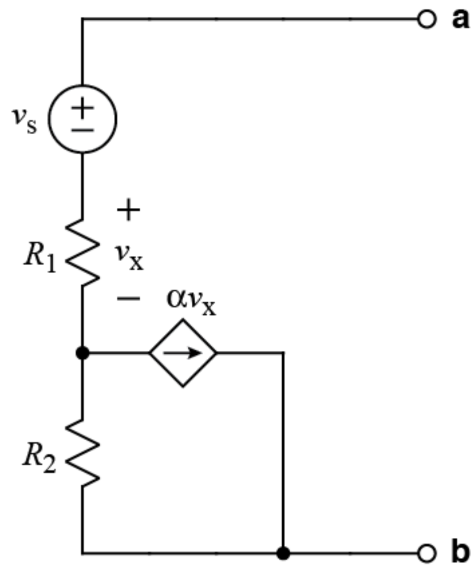
Ex:

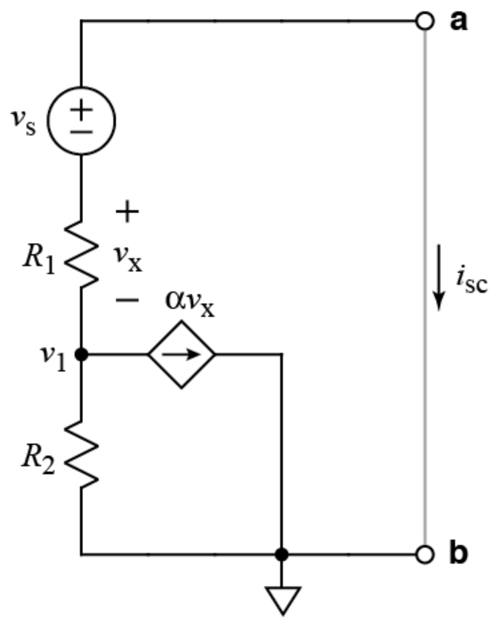


Find the Thevenin equivalent circuit at terminals **a** and **b**. v_x must not appear in your solution. **Note:** $\alpha \neq 0$.

SOL'N: The Thevenin equivalent voltage is v_{ab} across the **a** and **b** terminals with nothing connected at **a** and **b**, with the + sign at **a**. We observe that, if nothing is connected across **a** and **b**, no current flows in the left side of the circuit, and the voltage drop, v_x, across R_1 equals zero volts. This in turn means the dependent source turns into an open circuit. Since no current flows in the dependent source and in R_1, no current flows in R_2 and the voltage across R_2 is zero. Thus, the voltage across the **a** and **b** terminals,(i.e., the Thevenin equivalent voltage, v_{Th}), is v_s.

$$v_{Th} = v_s$$

Now we short out the **a** and **b** terminals to find current i_{sc} flowing from **a** to **b**. We use the node-voltage method, defining a node at the junction of the R's and dependent source.



We first define v_x in terms of the node voltage.

$$v_x = -v_s - v_1$$

Now we sum currents out of the node.

$$\frac{v_1 - -v_s}{R_1} + \alpha(-v_s - v_1) + \frac{v_1}{R_2} = 0\ \mathrm{A}$$

or

$$v_1\left(\frac{1}{R_1} - \alpha + \frac{1}{R_2}\right) = -\frac{v_s}{R_1} + \alpha v_s = v_s\left(\alpha - \frac{1}{R_1}\right)$$

or

$$v_1 = v_s \frac{\alpha - \dfrac{1}{R_1}}{\dfrac{1}{R_1} - \alpha + \dfrac{1}{R_2}} = v_s \frac{\alpha R_1 - 1}{1 - \alpha R_1 + \dfrac{R_1}{R_2}}$$

Now we observe that i_{sc} is the same as the current through R_1.

$$i_{sc} = \frac{v_1 - -v_s}{R_1} = \frac{v_s}{R_1}\left(\frac{\alpha R_1 - 1}{1 - \alpha R_1 + \dfrac{R_1}{R_2}} + 1\right)$$

or

$$i_{sc} = \frac{v_1 - -v_s}{R_1} = \frac{v_s}{R_1}\left(\frac{\alpha R_1 - 1}{1-\alpha R_1 + \frac{R_1}{R_2}} + \frac{1-\alpha R_1 + \frac{R_1}{R_2}}{1-\alpha R_1 + \frac{R_1}{R_2}}\right)$$

or

$$i_{sc} = \frac{v_s}{R_1}\left(\frac{\frac{R_1}{R_2}}{1-\alpha R_1 + \frac{R_1}{R_2}}\right) = v_s\left(\frac{\frac{1}{R_2}}{1-\alpha R_1 + \frac{R_1}{R_2}}\right) = v_s\left(\frac{1}{R_2 - \alpha R_1 R_2 + R_1}\right)$$

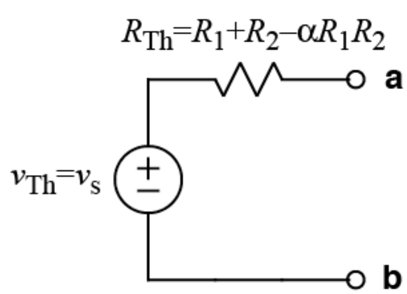
or

$$i_{sc} = v_s\left(\frac{1}{R_1 + R_2 - \alpha R_1 R_2}\right)$$

Now we use v_{Th} and i_{sc} to find the Thevenin equivalent resistance, R_{Th}.

$$R_{Th} = \frac{v_s}{i_{sc}} = \frac{v_s}{v_s\left(\frac{1}{R_1 + R_2 - \alpha R_1 R_2}\right)} = R_1 + R_2 - \alpha R_1 R_2$$

Thevenin equivalent:



NOTE: An alternative approach is to use a current divider to find an equivalent resistance for the dependent source. The current through R_1 splits into two parts, and we can equate the current in the dependent source with the current that would flow in an equivalent resistance, R_{eq}.

$$\frac{v_x}{R_1}\frac{R_2}{R_2 + R_{eq}} = \alpha v_x$$

Solving for R_{eq}, we have a value that is free of v_x.

$$R_{eq} = R_2\left(\frac{1}{\alpha R_1} - 1\right)$$

We may then replace the dependent source with R_{eq} and calculate R_{Th} as the resistance seen looking into the circuit from **a** and **b**.

$$R_{Th} = R_1 + R_2 \parallel R_{eq} = R_1 + R_2 \parallel \left[R_2\left(\frac{1}{\alpha R_1} - 1\right)\right]$$

or

$$R_{Th} = R_1 + R_2\left(1 \parallel \left(\frac{1}{\alpha R_1} - 1\right)\right) = R_1 + R_2\,\frac{\frac{1}{\alpha R_1} - 1}{1 + \frac{1}{\alpha R_1} - 1}$$

or

$$R_{Th} = R_1 + R_2\,\frac{\frac{1}{\alpha R_1} - 1}{\frac{1}{\alpha R_1}}$$

or

$$R_{Th} = R_1 + R_2(1 - \alpha R_1)$$